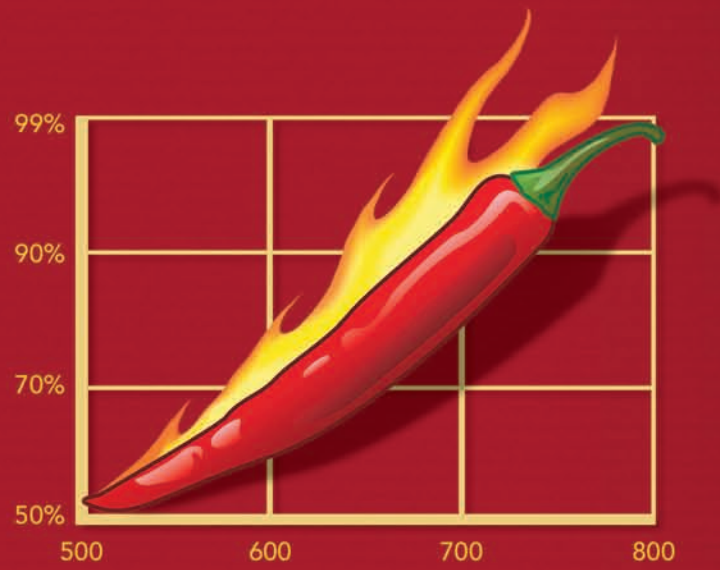
GMAT



PROBLEM SOLVING

BRANDON ROYAL

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CONTENTS

TOPICAL CHECKLIST

The following checklist provides an overview of all topical areas within each chapter. Reviewers may find it useful to check boxes upon completing each topic.

MATH:

CHAPTER 2

PROBLEM SOLVING

Math Class is tough.
—Barbie's original voice chip by Mattell

OVERVIEW

Official Exam Instructions for Problem Solving

Directions

Solve the problem and indicate the best of the answer choices given.

Numbers

All numbers used are real numbers.

Figures

A figure accompanying a problem solving question is intended to provide information useful in solving the problem. Figures are drawn as accurately as possible EXCEPT when it is stated in a specific problem that the figure is not drawn to scale. Straight lines may sometimes appear jagged. All figures lie in a plane unless otherwise indicated.

Strategies and Approaches

The following is a 4-step approach for math Problem Solving.

1. *Identify the type of problem and the appropriate math principle behind the problem at hand.*

   There are many different types of math problems on the GMAT. Each math problem in this book comes with a *classification* to highlight what category the problem belongs to and a *snapshot* to highlight why that particular problem was chosen, as well as any special problem-solving approach or math principle that is deemed relevant.

2. *Decide which approach to use to solve the problem—algebra, picking numbers, backsolving, approximation, or eyeballing.*

   There are both direct problem-solving approaches and indirect problem-solving approaches. The direct or algebraic approach involves applying actual math principles or formulas. Because we may not always know the correct algebraic method, we need an indirect or alternative approach. Other times, an indirect approach is plainly easier to apply than the algebraic approach. There are four alternative approaches for Problem Solving and these include: picking numbers, backsolving, approximation, and eyeballing.

3. *After performing calculations, always check again for what is being asked for.*

   Avoid making reading comprehension errors on the math section. Always re-read the question before choosing an answer, particularly if you have been engrossed in performing a longer computation.

4. *Employ elimination or guessing strategies, if necessary and when possible.*

   Guess if you must but employ guessing or elimination techniques.

Here are examples of each of the four indirect or alternative problem-solving approaches including guessing/elimination techniques.

i) Picking Numbers

If a and b are even integers, which of the following is an odd integer?

A) $ab + 2$
B) $a(b - 1)$
C) $a(a + 5)$
D) $3a + 4b$
E) $(a + 3)(b - 1)$

Choice E. This key strategy involves first picking numbers and then substituting them into the answer choices. Whenever a problem involves variables, we may consider using this strategy. For this particular problem, pick the numbers $a = 2$ and $b = 4$ because both are even integers, yet both are still small and manageable numbers. Now substitute. Answer choice E is correct. You can be confident that if it works for your chosen set of numbers, it will also work for all other numbers as well. There is no need to try other numbers.

$ab + 2$	$(2 \times 4) + 2 = 10$ even
$a(b - 1)$	$2(4 - 1) = 6$ even
$a(a + 5)$	$2(2 + 5) = 14$ even
$3a + 4b$	$(3 \times 2) + (4 \times 4) = 22$ even
$(a + 3)(b - 1)$	$(2 + 3)(4 - 1) = 15$ odd

ii) Backsolving

If $(x+2)^2 = -4 + 10x$, then which of the following could be the value of x?

A) 2
B) 1
C) 0
D) –1
E) –2

Choice A. The key to using backsolving is to use the answer choices and see if they work. In this respect, backsolving is like picking numbers except that the numbers we pick are one or more of the actual answer choices. Look for the answer which makes both sides equal. In this particular problem, we may choose to start testing on any single answer choice. Choice A is as good a starting point as any.

Choice A.
$(2 + 2)^2 = -4 + 10(2)$
$16 = 16$; This is the correct answer since both sides are equal.

Choice B.
$(1 + 2)^2 = -4 + 10(1)$; $9 = 6$. This is a wrong answer since both sides are not equal.

Choice C.
$(0 + 2)^2 = -4 + 10(0)$; $4 = -4$. This is a wrong answer since both sides are not equal.

Choice D.
$(-1 + 2)^2 = -4 + 10(-1)$; $1 = -14$. This is a wrong answer since both sides are not equal.

Choice E.
$(-2 + 2)^2 = -4 + 10(-2)$; $0 = -24$. This is a wrong answer since both sides are not equal.

iii) Approximation

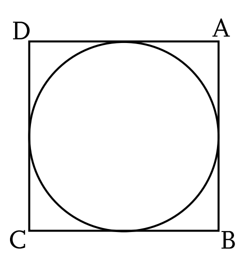
Approximately what percentage of the world's forested area is represented by Finland given that Finland has 53.42 million hectares of forested land of the world's 8.076 billion hectares of forested land.

A) 0.0066%
B) 0.066%
C) 0.66%
D) 6.6%
E) 66%

Choice C. Approximation is a strategy that helps us arrive at less than an exact number and the inclusion in this problem of the word "approximately" is an obvious clue. First, 8.076 billion is 8,076 million. Next, 8,076 million rounds to 8,000 million and 53.42 million rounds to 53 million. Dividing 53 million by 8,000 million we arrive at 0.0066 (53M/8,000M). We convert this decimal figure to a percentage by multiplying by 100 (or moving the decimal point two places to the right) and adding a percent sign in order to obtain our answer of 0.66%. Note that the shortcut method involves comparing 53 million to 1% of 8,000 million or 80 million. Since 53 million is approximately two-thirds of 80 million then the answer is some two-thirds of 1% or 0.66%.

iv) Eyeballing

If the figure below is a square with a side of 4 units, what is the area of the enclosed circle, expressed to the nearest whole number?



A) π
B) 4
C) 8
D) 13
E) 16

Choice D. Eyeballing is a parallel technique to be used on diagrams. Note that whatever the area of this circle may be, it must be less than the area of this square. The area of the square (in square units) is: $A = s^2 = 4 \times 4 = 16$. Therefore, the area of the circle is a little less than 16. Choice D is the only close answer. For the record, the near exact area of the circle is: $A = \pi r^2 = 3.14(2)^2 = 12.56$ or 13. Note that the decimal approximation for π is 3.14 while the fractional approximation is $\frac{22}{7}$.

v) Elimination and Guessing

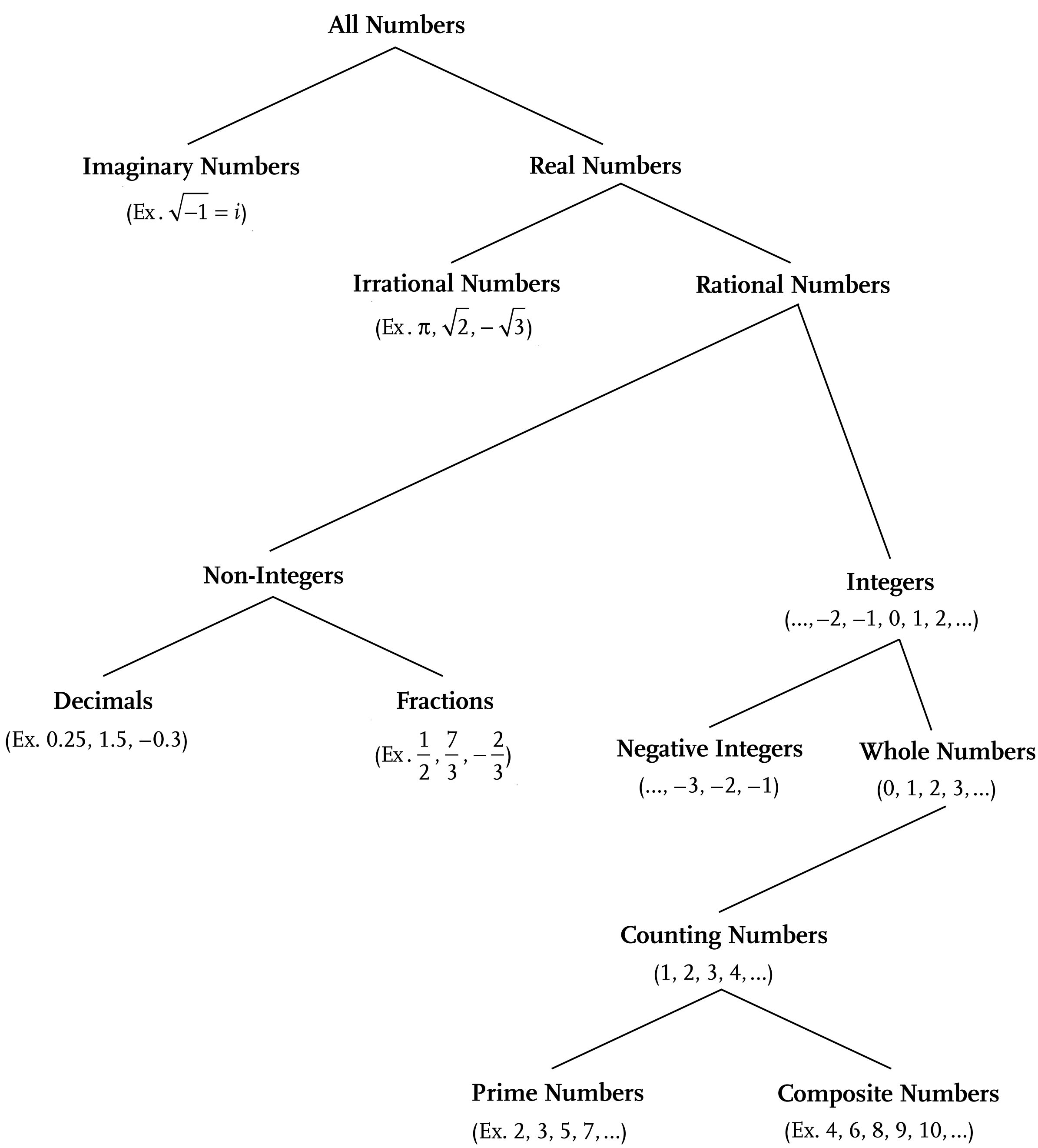
A broker invested her own money in the stock market. During the first year, she increased her stock market wealth by 50 percent. In the second year, largely as a result of a slump in the stock market, she suffered a 30 percent decrease in the value of her stock investments. What was the net increase or decrease on her overall stock investment wealth by the end of the second year?

A) –5%
B) 5%
C) 15%
D) 20%
E) 80%

Choice B. If you must guess, the key strategies of elimination include: (1) eliminate an answer that looks different from the others (2) eliminate answers which look too big or too small, i.e., extreme answers, and (3) eliminate answers which contain the same or similar numbers as given in the question or are easy derivatives of the numbers used in the problem. By easy derivatives, think in terms of addition and subtraction, not multiplication and division. For example, eliminate –5% because it is negative, and thus different from the other positive numbers. Eliminate 80% because it is much bigger than any other number (extreme). Eliminate 20% because it is an easy derivative of the numbers mentioned in the question, (i.e., 50% less 30%). You would then guess choices B or C. The actual answer is obtained by multiplying 150% by 70% and subtracting 100% from this total. That is: 150% × 70% = 105%; 105% – 100% = 5%.

REVIEW OF BASIC MATH

Flowcharting the World of Numbers



Note: Imaginary numbers are not tested on the GMAT.

Numbers are first divided into real and imaginary numbers. *Imaginary* numbers are not a part of everyday life. *Real* numbers are further divided into rational and irrational numbers. *Irrational* numbers are numbers which cannot be expressed as simple integers, fractions, or decimals; non-repeating decimals are always irrational numbers of which π may be the most famous. *Rational* numbers include integers and non-integers. *Integers* are positive and negative whole numbers while *non-integers* include decimals and fractions.

Number Definitions

Real numbers: Any number which exists on the number line. Real numbers are the combined group of rational and irrational numbers.

Imaginary numbers: Any number multiplied by *i*, the imaginary unit: $i = \sqrt{-1}$. Imaginary numbers are the opposite of real numbers and are not part of our everyday life.

Rational numbers: Numbers which can be expressed as a fraction whose top (the numerator) and bottom (the denominator) are both integers.

Irrational numbers: Numbers which can't be expressed as a fraction whose top (the numerator) and bottom (the denominator) are integers. Square roots of non-perfect squares (such as $\sqrt{2}$) are irrational, and π is irrational. Irrational numbers may be described as non-repeating decimals.

Integers: Integers consist of those numbers which are multiples of 1: {..., –2, –1, 0, 1, 2, 3, ...}. Integers are "integral"—they contain no fractional or decimal parts.

Non-integers: Non-integers are those numbers which contain fractional or decimal parts. E.g., $\frac{1}{2}$ and 0.125 are non-integers.

Whole numbers: Non-negative integers: {0, 1, 2, 3, ...}. Note that 0 is a whole number (but a non-negative whole number, since it is neither positive nor negative).

Counting numbers: The subset of whole numbers which excludes 0: {1, 2, 3, ...}.

Prime numbers: Prime numbers are a subset of the counting numbers. They include those non-negative integers which have two and only two factors; that is, the factors 1 and themselves. The first 10 primes are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29. Note that 1 is not a prime number as it only has one factor, i.e., 1. Also, the number 2 is not only the smallest prime but also the only even prime number.

Composite numbers: A positive number that has more than two factors other than 1 and itself. Also, any non-prime number greater than 1. Examples include: 4, 6, 8, 9, 10, etc. Note that 1 is not a composite number and the number 4 is the smallest composite number.

Factors: A factor is an integer that can be divided evenly into another integer ("divided evenly" means that there is no remainder). For example, the factors of 12 are 1, 2, 3, 4, 6, and 12.

Multiples: A multiple is a number that results from a given integer being multiplied by another integer. Example: Multiples of 12 include 12, 24, 36, 48, etc. Proof: $12 \times 1 = 12$, $12 \times 2 = 24$, $12 \times 3 = 36$, and $12 \times 4 = 48$, etc. Note that whereas a factor of any number is less than or equal to the number in question, a multiple of any number is equal to or greater than the number itself. That is, any non-zero integer has a finite number of factors but an infinite number of multiples.

The Four Basic Operations

The four basic arithmetic operations are addition, subtraction, multiplication, and division. The results of these operations are called sum, difference, product, and quotient, respectively. Two additional operations involve exponents and radicals.

Addition

2	augend
+3	addend
5	sum

Subtraction

25	minuend
−16	subtrahend
9	difference

Multiplication

3	multiplicand
×4	multiplier
12	product

Division

10	dividend
÷5	divisor
2	quotient

Exponents

	power
2^3	base
$= 8$	result

Radicals

	radical
$\sqrt{9}$	radicand
$= 3$	root

Common Fractions and their Percentage Equivalents

Exercise – Fill in the missing percentages to change common fractions into their percentage equivalents.

$\frac{1}{1} = 100\%$

$\frac{1}{2} = 50\%$ $\frac{2}{2} = 100\%$

$\frac{1}{3} = ?\%$ $\frac{2}{3} = ?\%$ $\frac{3}{3} = 100\%$

$\frac{1}{4} = ?\%$ $\frac{2}{4} = 50\%$ $\frac{3}{4} = ?\%$ $\frac{4}{4} = 100\%$

$\frac{1}{5} = ?\%$ $\frac{2}{5} = ?\%$ $\frac{3}{5} = ?\%$ $\frac{4}{5} = ?\%$ $\frac{5}{5} = 100\%$

$\frac{1}{6} = ?\%$ $\frac{2}{6} = ?\%$ $\frac{3}{6} = 50\%$ $\frac{4}{6} = ?\%$ $\frac{5}{6} = ?\%$ $\frac{6}{6} = 100\%$

$\frac{1}{7} = ?\%$ $\frac{2}{7} = ?\%$ $\frac{3}{7} = ?\%$ $\frac{4}{7} = ?\%$ $\frac{5}{7} = ?\%$ $\frac{6}{7} = ?\%$ $\frac{7}{7} = 100\%$

$\frac{1}{8} = ?\%$ $\frac{2}{8} = ?\%$ $\frac{3}{8} = ?\%$ $\frac{4}{8} = 50\%$ $\frac{5}{8} = ?\%$ $\frac{6}{8} = ?\%$ $\frac{7}{8} = ?\%$ $\frac{8}{8} = 100\%$

$\frac{1}{9} = ?\%$ $\frac{2}{9} = ?\%$ $\frac{3}{9} = ?\%$ $\frac{4}{9} = ?\%$ $\frac{5}{9} = ?\%$ $\frac{6}{9} = ?\%$ $\frac{7}{9} = ?\%$ $\frac{8}{9} = ?\%$ $\frac{9}{9} = 100\%$

$\frac{1}{10} = ?\%$ $\frac{2}{10} = ?$ $\frac{3}{10} = ?\%$ $\frac{4}{10} = ?$ $\frac{5}{10} = 50\%$ $\frac{6}{10} = ?$ $\frac{7}{10} = ?$ $\frac{8}{10} = ?$ $\frac{9}{10} = ?$ $\frac{10}{10} = 100\%$

Solutions – Common fractions and their percentage equivalents.

$\frac{1}{1} = 100\%$

$\frac{1}{2} = 50\%$ $\quad \frac{2}{2} = 100\%$

$\frac{1}{3} = 33\frac{1}{3}\%$ $\quad \frac{2}{3} = 66\frac{2}{3}\%$ $\quad \frac{3}{3} = 100\%$

$\frac{1}{4} = 25\%$ $\quad \frac{2}{4} = 50\%$ $\quad \frac{3}{4} = 75\%$ $\quad \frac{4}{4} = 100\%$

$\frac{1}{5} = 20\%$ $\quad \frac{2}{5} = 40\%$ $\quad \frac{3}{5} = 60\%$ $\quad \frac{4}{5} = 80\%$ $\quad \frac{5}{5} = 100\%$

$\frac{1}{6} = 16\frac{2}{3}\%$ $\quad \frac{2}{6} = 33\frac{1}{3}\%$ $\quad \frac{3}{6} = 50\%$ $\quad \frac{4}{6} = 66\frac{2}{3}\%$ $\quad \frac{5}{6} = 83\frac{1}{3}\%$ $\quad \frac{6}{6} = 100\%$

$\frac{1}{7} = 14.3\%$ $\quad \frac{2}{7} = 28.6\%$ $\quad \frac{3}{7} = 42.9\%$ $\quad \frac{4}{7} = 57.1\%$ $\quad \frac{5}{7} = 71.4\%$ $\quad \frac{6}{7} = 85.7\%$ $\quad \frac{7}{7} = 100\%$

$\frac{1}{8} = 12.5\%$ $\quad \frac{2}{8} = 25\%$ $\quad \frac{3}{8} = 37.5\%$ $\quad \frac{4}{8} = 50\%$ $\quad \frac{5}{8} = 62.5\%$ $\quad \frac{6}{8} = 75\%$ $\quad \frac{7}{8} = 87.5\%$ $\quad \frac{8}{8} = 100\%$

$\frac{1}{9} = 11.11\%$ $\quad \frac{2}{9} = 22.22\%$ $\quad \frac{3}{9} = 33.33\%$ $\quad \frac{4}{9} = 44.44\%$ $\quad \frac{5}{9} = 55.55\%$ $\quad \frac{6}{9} = 66.66\%$ $\quad \frac{7}{9} = 77.77\%$ $\quad \frac{8}{9} = 88.88\%$ $\quad \frac{9}{9} = 100\%$

$\frac{1}{10} = 10\%$ $\quad \frac{2}{10} = 20\%$ $\quad \frac{3}{10} = 30\%$ $\quad \frac{4}{10} = 40\%$ $\quad \frac{5}{10} = 50\%$ $\quad \frac{6}{10} = 60\%$ $\quad \frac{7}{10} = 70\%$ $\quad \frac{8}{10} = 80\%$ $\quad \frac{9}{10} = 90\%$ $\quad \frac{10}{10} = 100\%$

Rules for Odd and Even Numbers

	Scenario 1:	*Scenario 2:*
Even + Even = Even	$2 + 2 = 4$	$-2 + -2 = -4$
Odd + Odd = Even	$3 + 3 = 6$	$-3 + -3 = -6$
Even + Odd = Odd	$2 + 3 = 5$	$-2 + -3 = -5$
Odd + Even = Odd	$3 + 2 = 5$	$-3 + -2 = -5$
Even – Even = Even	$4 - 2 = 2$	$-4 - (-2) = -2$
Odd – Odd = Even	$5 - 3 = 2$	$-5 - (-3) = -2$
Even – Odd = Odd	$6 - 3 = 3$	$-6 - (-3) = -3$
Odd – Even = Odd	$5 - 2 = 3$	$-5 - (-2) = -3$
Even × Even = Even	$2 \times 2 = 4$	$-2 \times -2 = 4$
Odd × Odd = Odd	$3 \times 3 = 9$	$-3 \times -3 = 9$
Even × Odd = Even	$2 \times 3 = 6$	$-2 \times -3 = 6$
Odd × Even = Even	$3 \times 2 = 6$	$-3 \times -2 = 6$
Even ÷ Even = Even	$4 \div 2 = 2$	$-4 \div -2 = 2$
Odd ÷ Odd = Odd	$9 \div 3 = 3$	$-9 \div -3 = 3$
Even ÷ Odd = Even	$6 \div 3 = 2$	$-6 \div -3 = 2$
Odd ÷ Even = *Not Possible	$5 \div 2 = 2\frac{1}{2}$	$-5 \div -2 = 2\frac{1}{2}$

* With respect to the last example above, an odd number divided by an even number does not result in either an even or odd integer; it results in a non-integer.

Rules for Positive and Negative Numbers

	Scenario 1:	*Scenario 2:*
Positive + Positive = Positive	$2 + 2 = 4$	
Negative + Negative = Negative	$-2 + (-2) = -4$	
Positive + Negative = Depends	$4 + (-2) = 2$	$2 + (-4) = -2$
Negative + Positive = Depends	$-2 + 4 = 2$	$-4 + 2 = -2$
Positive – Positive = Depends	$4 - 2 = 2$	$2 - 4 = -2$
Negative – Negative = Depends	$-2 - (-4) = 2$	$-4 - (-2) = -2$
Positive – Negative = Positive	$2 - (-2) = 4$	
Negative – Positive = Negative	$-4 - 2 = -6$	
Positive × Positive = Positive	$2 \times 2 = 4$	
Negative × Negative = Positive	$-2 \times -2 = 4$	
Positive × Negative = Negative	$2 \times -2 = -4$	
Negative × Positive = Negative	$-2 \times 2 = -4$	
Positive ÷ Positive = Positive	$4 \div 2 = 2$	
Negative ÷ Negative = Positive	$-4 \div -2 = 2$	
Positive ÷ Negative = Negative	$4 \div -2 = -2$	
Negative ÷ Positive = Negative	$-4 \div 2 = -2$	

Common Squares, Cubes, and Square Roots

$1^2 = 1$	$1^3 = 1$	$1^4 = 1$	$1^5 = 1$	$1^6 = 1$
$2^2 = 4$	$2^3 = 8$	$2^4 = 16$	$2^5 = 32$	$2^6 = 64$
$3^2 = 9$	$3^3 = 27$	$3^4 = 81$	$3^5 = 243$	
$4^2 = 16$	$4^3 = 64$	$4^4 = 256$		
$5^2 = 25$	$5^3 = 125$	$5^4 = 625$		
$6^2 = 36$	$6^3 = 216$			
$7^2 = 49$	$7^3 = 343$			
$8^2 = 64$	$8^3 = 512$			
$9^2 = 81$	$9^3 = 729$			

Also:

$10^2 = 100$
$10^3 = 1{,}000$
$10^4 = 10{,}000$
$10^5 = 100{,}000$
$10^6 = 1{,}000{,}000$
$10^9 = 1$ billion
$10^{12} = 1$ trillion

Note: In most English-speaking countries today (particularly the U.S., Great Britain, Canada, and Australia), one billion equals 1,000,000,000 or 10^9, or one thousand millions. In many other countries including France, Germany, Spain, Norway, and Sweden, the word "billion" indicates 10^{12}, or one million millions. Although Britain and Australia have traditionally employed the international usage of 10^{12}, they have now largely switched to the U.S. version of 10^9.

In short, for GMAT purposes, a billion equals 10^9 and a trillion equals 10^{12}.

Common Squares from 13 to 30

$13^2 = 169$	$19^2 = 361$	$25^2 = 625$
$14^2 = 196$	$20^2 = 400$	$26^2 = 676$
$15^2 = 225$	$21^2 = 441$	$27^2 = 729$
$16^2 = 256$	$22^2 = 484$	$28^2 = 784$
$17^2 = 289$	$23^2 = 529$	$29^2 = 841$
$18^2 = 324$	$24^2 = 576$	$30^2 = 900$

Common Square Roots

$\sqrt{1} = 1$

$\sqrt{2} = 1.4$

$\sqrt{3} = 1.7$

$\sqrt{4} = 2$

$\sqrt{5} = 2.2$

Pop Quiz

See pages 84–85 for solutions.

Review – Fractions to Percents

Convert the following fractions to their percentage equivalents:

$\frac{1}{3} =$	$\frac{2}{3} =$	$\frac{1}{6} =$	$\frac{5}{6} =$	$\frac{1}{8} =$
$\frac{3}{8} =$	$\frac{5}{8} =$	$\frac{7}{8} =$	$\frac{1}{9} =$	$\frac{5}{9} =$

Review – Decimals to Fractions

Convert the following decimals, which are greater than 1, into fractional equivalents.

1.2 =   1.25 =   1.33 =

Simplify each expression below without multiplying decimals or using a calculator. (Hint: Translate each decimal into a common fraction and then multiply and/or divide.)

$(1.25)(0.50)(0.80)(2.00) =$

$\frac{(0.7500)(0.8333)}{(0.6250)} =$

$\frac{(0.2222)}{(0.3333)(0.6666)} =$

Review – Common Squares from 13 to 30

Fill in the missing numbers to complete the Pythagorean triplets below. A Pythagorean triple consists of three positive integers *a, b,* and *c,* such that $a^2 + b^2 = c^2$.

3 : 4 : 5 5 : 12 : __ 7 : __ : 25 8 : 15 : __

Review – Common Square Roots

Put the following statements in order, from largest to smallest value. (Hint: Approximate each square root to one decimal point.)

I. $1 + \sqrt{5}$
II. $2 + \sqrt{3}$
III. $3 + \sqrt{2}$

Review – Exponents and Radicals

Put the following statements in order, from largest to smallest value.

I. 4
II. 4^2
III. $\sqrt{4}$
IV. $\frac{1}{4}$
V. $\left(\frac{1}{4}\right)^2$
VI. $\sqrt{\frac{1}{4}}$

Divisibility Rules

No.	Divisibility Rule	Examples
1	Every number is divisible by 1.	15 divided by 1 equals 15.
2	A number is divisible by 2 if it is even.	24 divided by 2 equals 12.
3	A number is divisible by 3 if the sum of its digits is divisible by 3.	651 is divisible by 3 since 6 + 5 + 1 = 12 and "12" is divisible by 3.
4	A number is divisible by 4 if its last two digits form a number that is divisible by 4.	1,112 is divisible by 4 since the number "12" is divisible by 4.
5	A number is divisible by 5 if the number ends in 5 or 0.	245 is divisible by 5 since this number ends in 5.
6	A number is divisible by 6 if it is divisible by both 2 and 3.	738 is divisible by 6 since this number is divisible by both 2 and 3, and the rules that govern the divisibility of 2 and 3 apply.
7	No clear rule.	N.A.
8	A number is divisible by 8 if its last three digits form a number that is divisible by 8.	2,104 is divisible by 8 since the number "104" is divisible by 8.
9	A number is divisible by 9 if the sum of its digits is divisible by 9.	4,887 is divisible by 9 since 4 + 8 + 8 + 7 = 27 and 27 is divisible by 9.
10	A number is divisible by 10 if it ends in 0.	990 is divisible by 10 because 990 ends in 0.

Exponents

Here are the ten basic rules governing exponents:

Rule 1 $a^b \times a^c = a^{b+c}$

Example $2^2 \times 2^2 = 2^{2+2} = 2^4$

Rule 2 $a^b \div a^c = a^{b-c}$

Example $2^6 \div 2^2 = 2^{6-2} = 2^4$

Rule 3 $(a^b)^c = a^{b \times c}$

Example $(2^2)^3 = 2^{2 \times 3} = 2^6$

Rule 4 $(ab)^c = a^c b^c$

Example $6^2 = (2 \times 3)^2 = 2^2 \times 3^2$

Rule 5 $\frac{a^c}{b^c} = \left(\frac{a}{b}\right)^c$

Example $\frac{4^5}{2^5} = \left(\frac{4}{2}\right)^5 = 2^5$

Rule 6 $a^{-b} = \frac{1}{a^b}$

Example $2^{-3} = \frac{1}{2^3}$

Rule 7 i) $a^{1/2} = \sqrt{a}$

Example $(4)^{1/2} = \sqrt{4} = 2$

ii) $a^{1/3} = \sqrt[3]{a}$

Example $(27)^{1/3} = \sqrt[3]{27} = 3$

iii) $a^{2/3} = \left(\sqrt[3]{a}\right)^2$

Example $(64)^{2/3} = \left(\sqrt[3]{64}\right)^2 = 4^2 = 16$

Rule 8 $a^b + a^b = a^b(1+1) = a^b(2) = 2a^b$

Example $2^{10} + 2^{10} = 2^{10}(1+1) = 2^{10}(2) = 2^{10}(2^1) = 2^{11}$

Rule 9 $a^b + a^c \neq a^{b+c}$

Example $2^2 + 2^3 \neq 2^{2+3}$

Rule 10 $a^b - a^c \neq a^{b-c}$

Example $2^5 - 2^2 \neq 2^{5-2}$

Radicals

Here are the ten basic rules governing radicals:

Rule 1 $\left(\sqrt{a}\right)^2 = a$

Example $\left(\sqrt{4}\right)^2 = 4$

Proof $\sqrt{4} \times \sqrt{4} = \sqrt{16} = 4$

Rule 2 $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$

Example $\sqrt{4} \times \sqrt{9} = \sqrt{36}$

Proof $\sqrt{4} = 2; \sqrt{9} = 3.\ \text{Thus},\ 2 \times 3 = 6$

Rule 3 $\sqrt{\text{a}} \div \sqrt{\text{b}} = \sqrt{\frac{a}{b}}$

Example $\sqrt{100} \div \sqrt{25} = \sqrt{4}$

Proof $\sqrt{100} = 10; \sqrt{25} = 5.\ \text{Thus},\ 10 \div 5 = 2$

Rule 4 $\frac{\sqrt[c]{a}}{\sqrt[c]{b}} = \sqrt[c]{\frac{a}{b}}$

Example $\frac{\sqrt[3]{64}}{\sqrt[3]{8}} = \sqrt[3]{\frac{64}{8}}$

Proof $\sqrt[3]{64} = 4; \sqrt[3]{8} = 2.\ \text{Thus},\ 4 \div 2 = 2$

Rule 5 $b\sqrt{a} + c\sqrt{a} = (b + c)\sqrt{a}$

Example $3\sqrt{4} + 2\sqrt{4} = 5\sqrt{4}$

Proof $\sqrt{4} = 2.\ \text{Thus},\ 3(2) + 2(2) = 5(2)$

Rule 6 $b\sqrt{a} - c\sqrt{a} = (b - c)\sqrt{a}$

Example $5\sqrt{9} - 2\sqrt{9} = 3\sqrt{9}$

Proof $\sqrt{9} = 3.\ \text{Thus},\ 5(3) - 2(3) = 3(3)$

Rule 7 $\quad \dfrac{b}{\sqrt{a}} = \dfrac{b}{\sqrt{a}} \times \dfrac{\sqrt{a}}{\sqrt{a}} = \dfrac{b\sqrt{a}}{a}$

Example $\quad \dfrac{6}{\sqrt{3}} = \dfrac{6}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{6\sqrt{3}}{\sqrt{9}} = \dfrac{6\sqrt{3}}{3} = 2\sqrt{3}$

In the calculation directly above, we multiply both the numerator and denominator of the original fraction by $\sqrt{3}$ $\left(\text{i.e., } \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}\right)$ in order to remove the radical from the denominator of this fraction.

Rule 8 $\quad \dfrac{\sqrt{a}+1}{\sqrt{a}-1} = \dfrac{\sqrt{a}+1}{\sqrt{a}-1} \times \dfrac{\sqrt{a}+1}{\sqrt{a}+1}$

Example $\quad \dfrac{\sqrt{2}+1}{\sqrt{2}-1} = \dfrac{\sqrt{2}+1}{\sqrt{2}-1} \times \dfrac{\sqrt{2}+1}{\sqrt{2}+1} = 3 + 2\sqrt{2}$

In the calculation directly above, we multiply both the numerator and denominator of the fraction by $\sqrt{2}+1$ $\left(\text{i.e., } \frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}\right)$ in order to remove the radical from the denominator of this fraction.

$$\frac{\sqrt{a}-1}{\sqrt{a}+1} = \frac{\sqrt{a}-1}{\sqrt{a}+1} \times \frac{\sqrt{a}-1}{\sqrt{a}-1}$$

Example $\quad \dfrac{\sqrt{2}-1}{\sqrt{2}+1} = \dfrac{\sqrt{2}-1}{\sqrt{2}+1} \times \dfrac{\sqrt{2}-1}{\sqrt{2}-1} = 3 - 2\sqrt{2}$

By multiplying both the numerator and denominator of the fraction by $\sqrt{2}-1$ $\left(\text{i.e., } \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}\right)$ we can remove the radical from the denominator of this fraction.

Rule 9 $\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$

Example $\sqrt{16} + \sqrt{9} \neq \sqrt{25}$

Proof $\sqrt{16} = 4; \sqrt{9} = 3.$ Thus, $4 + 3 \neq 5$

Rule 10 $\sqrt{a} - \sqrt{b} \neq \sqrt{a-b}$

Example $\sqrt{25} - \sqrt{16} \neq \sqrt{9}$

Proof $\sqrt{25} = 5; \sqrt{16} = 4.$ Thus, $5 - 4 \neq 3$

Basic Geometry Formulas

Circles

Circumference:

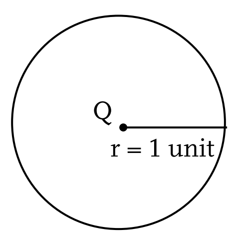
Circumference = $\pi \times$ diameter

$C = \pi d$ or $C = 2\pi r$
[where r = radius and Q = center point]

Area:

Area = $\pi \times \text{radius}^2$

$A = \pi r^2$



Triangles

Area:

$$\text{Area} = \frac{\text{base} \times \text{height}}{2}$$

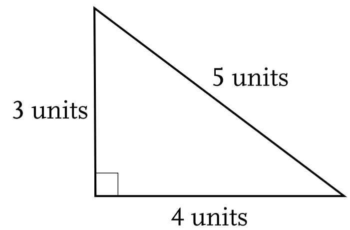
$$A = \frac{bh}{2}$$

The Pythagorean Theorem:

$c^2 = a^2 + b^2$
[where c is the length of the hypotenuse and a and b are the length of the legs]

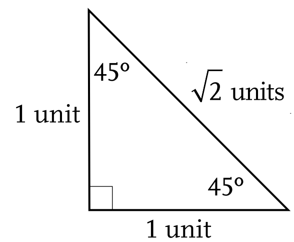
3 : 4 : 5 Triangle

In a $3:4:5$ triangle, the ratios of the length of the sides are always $3:4:5$ units.



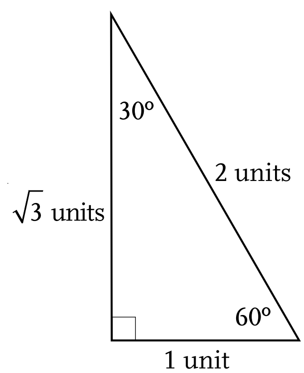
45°–45°–90° Triangle

In a 45°–45°–90° triangle, the ratios of the length of the sides are $1:1:\sqrt{2}$ units. A right-isosceles triangle is another name for a 45°–45°–90° triangle.



30°–60°–90° Triangle

In a 30°–60°–90° triangle, the ratios of the lengths of the sides are $1:\sqrt{3}:2$ units.



Squares

Perimeter:

$\text{Perimeter} = 4 \times \text{side}$

$P = 4s$

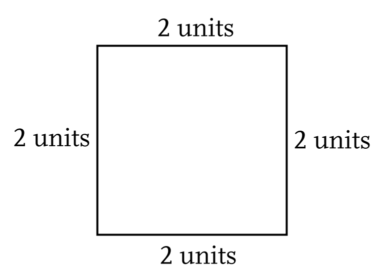
Example $\quad P = 4(2) = 8$ units

Area:

$\text{Area} = \text{side}^2$

$A = s^2$

Example $\quad A = (2)^2 = 4 \text{ units}^2$



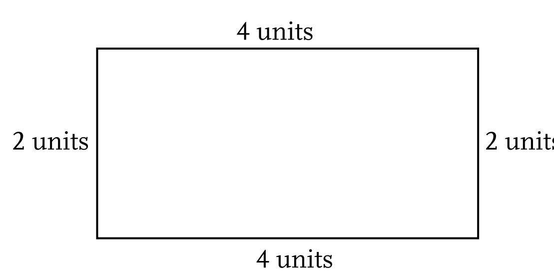
Rectangles

Perimeter:

$\text{Perimeter} = (2 \times \text{length}) + (2 \times \text{width})$

$P = 2l + 2w$

Example $\quad P = 2(4) + 2(2) = 12$ units



Area:

Area = length × width

$A = lw$

Example $A = 4 \times 2 = 8$ units^2

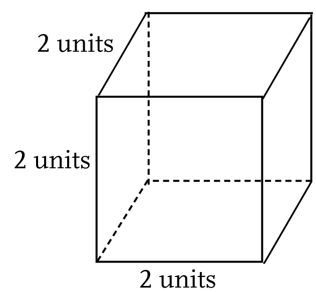
Cubes

Surface Area:

Surface Area = $6 \times \text{side}^2$

$SA = 6s^2$

Example $SA = 6(2)^2 = 24$ units^2



Volume:

Volume = side^3

$V = s^3$

Example $V = 2^3 = 8$ units^3

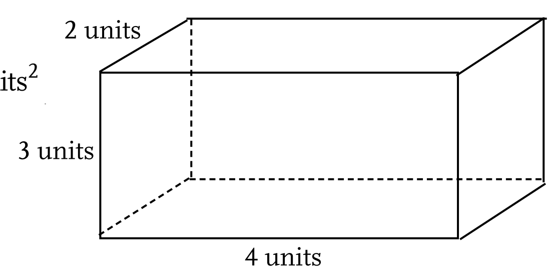
Rectangular Solids

Surface Area:

Surface Area = 2(length × width) + 2(length × heigth) + 2(width × height)

$SA = 2lw + 2lh + 2wh$

Example $SA = 2(4 \times 2) + 2(4 \times 3) + 2(2 \times 3) = 52$ units^2



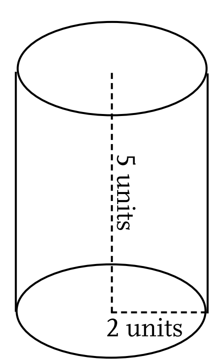
Volume:

Volume = length × width × height

$V = lwh$

Example $V = 4 \times 2 \times 3 = 24$ units^3

Circular Cylinders



Surface Area:

Surface Area = $2\pi(\text{radius})^2 + \pi(\text{diameter})(\text{height})$

Surface Area = $2\pi r^2 + \pi dh$

Example $SA = 2\pi(2)^2 + \pi(4)(5)$

$SA = 8\pi + 20\pi = 28\pi$ units^2

Volume:

Volume $= \pi r^2 h$

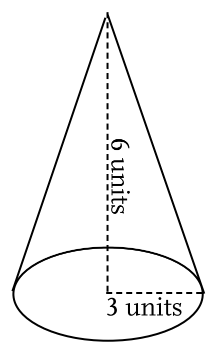
Example $V = \pi(2)^2(5) = 20\pi$ units^3

Cone

Volume:

Volume $= \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$

Example $V = \frac{1}{3}\pi(3)^2(6) = 18\pi$ units^3



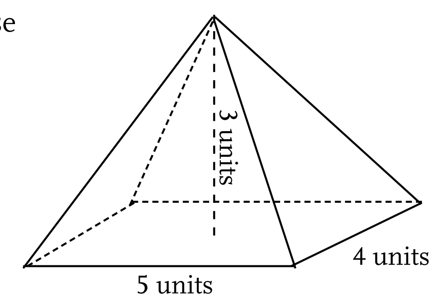
Pyramids

Volume:

Volume $= \frac{1}{3}Bh$

[where B is the area of the base and h is the height]

Example $V = \frac{1}{3}(20)(3)$ units^3



Note: The formula for all tapered

solids is the same: $V = \frac{1}{3}Bh$,

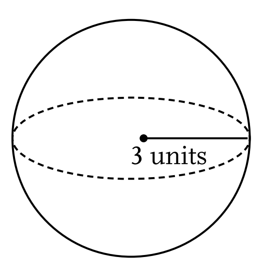
where B is the area of the base and h is the perpendicular distance from the base to the vertex.

Sphere

Surface Area:

Surface Area $= 4\pi r^2$

Example $SA = 4\pi(3)^2 = 36\pi$ units^2



Volume:

Volume $= \frac{4}{3}\pi r^3$

Example $V = \frac{4}{3}\pi(3)^3 = 36\pi$ units^3

Probability, Permutations & Combinations

Overview

Exhibits 2.1 & 2.2 are strategic flowcharts for use in both previewing and reviewing the material in this section. First, what is the difference between probability and permutations & combinations? Probabilities are expressed as fractions, percents, or decimals between 0 and 1 (where 1 is the probability of certainty and 0 is the probability of impossibility). Permutations and combinations, on the other hand, result in outcomes greater than or equal to 1. Frequently they result in quite large outcomes such as 10, 36, 720, etc.

In terms of probability, a quick rule of thumb is to determine first whether we are dealing with an "and" or "or" situation. "And" means multiply and "or" means addition. For example, if a problem states, "what is the probability of x and y," we multiply individual probabilities together. If a problem states, "what is the probability of x or y," we add individual probabilities together.

Moreover, if a probability problem requires us to multiply, we must ask one further question: are the events independent or are the events dependent? Independent means that two events have no influence on one another and we simply multiply individual probabilities together to arrive at a final answer. Dependent events mean that the occurrence of one event has an influence on the occurrence of another event, and this influence must be taken into account.

Likewise, if a problem requires us to add probabilities, we must ask one further question: are the events mutually exclusive or non-mutually exclusive? Mutually exclusive means that two events cannot occur at the same time and there is no "overlap" present. If two events have no overlap, we simply add probabilities. Non-mutually exclusive means that two events can occur at the same time and overlap is present. If two events do contain overlap, this overlap must not be double counted.

With respect to permutations and combinations, permutations are ordered groups while combinations are unordered groups. That is, order matters in permutations; order does not matter in combinations. For example, AB and BA are considered different outcomes in permutations but are considered a single outcome in combinations. In real-life, examples of permutations include telephone numbers, license plates, electronic codes, and passwords. Examples of combinations include selection of members for a team or lottery tickets. In the case of lottery tickets, for instance, the order of numbers does not matter; we just need to get all the numbers, usually six of them.

Note that in problem solving situations, the words "arrangements" or "possibilities" imply permutations; the words "select" or "choose" imply combinations.

Factorials:

Factorial means that we engage multiplication such that:

Example $4! = 4 \times 3 \times 2 \times 1$

Example $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

Zero factorial equals one and one factorial also equals one:

Example $0! = 1$

Example $1! = 1$

Coins, Cards, Dice, and Marbles:

Problems in this section include reference to coins, dice, marbles, and cards. For clarification purposes: The two sides of a coin are heads and tails. A die has six sides numbered from 1 to 6, with each having an equal likelihood of appearing subsequent to being tossed. The word "die" is singular; "dice" is plural. Marbles are assumed to be of a single, solid color. A deck of cards contains 52 cards divided equally into four suits—Clubs, Diamonds, Hearts, and Spades—where each suit contains 13 cards including Ace, King, Queen, Jack, 10, 9, 8, 7, 6, 5, 4, 3, and 2. Card problems have not appeared on the GMAT in recent years.

EXHIBIT 2.1 PROBABILITY FLOWCHART

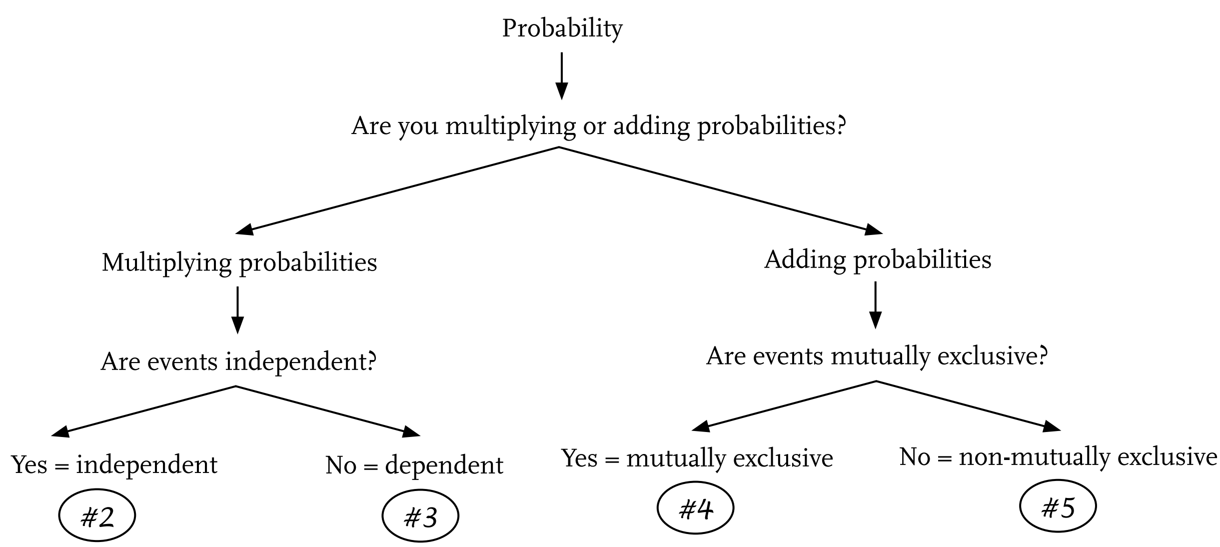
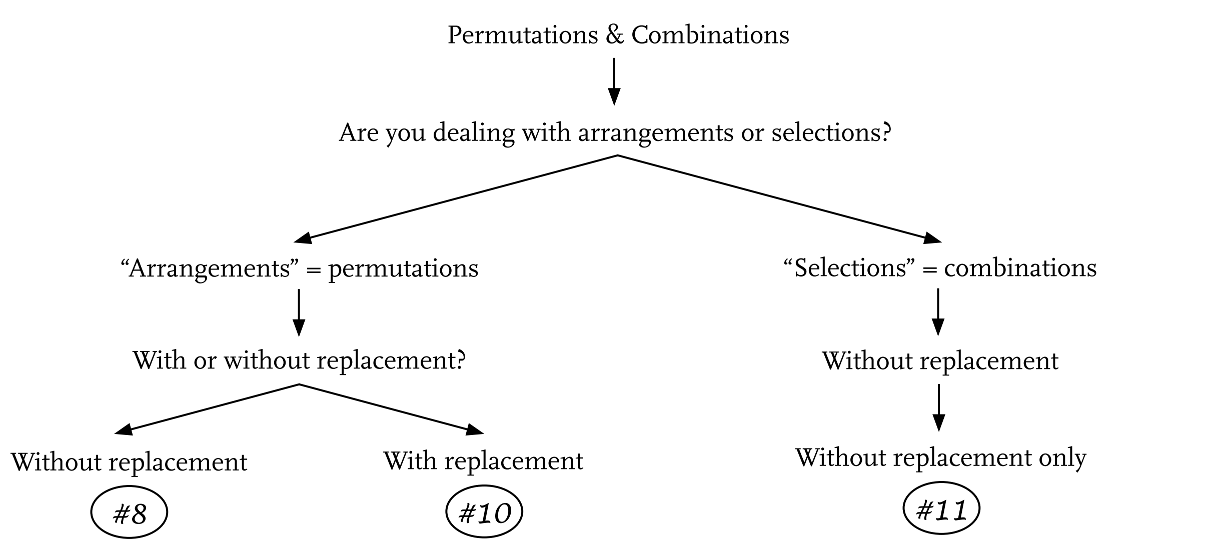


EXHIBIT 2.2 PERMUTATIONS AND COMBINATIONS FLOWCHART



Basic Probability Formulas

Here at a glance are the basic probability, permutation, and combination formulas used in this chapter and applicable to the GMAT.

Universal Formula

#1 $$\text{Probability} = \frac{\text{Selected Events(s)}}{\text{Total Number of Possibilities}}$$

Example You buy 3 raffle tickets and there are 10,000 tickets sold. What is the probability of winning the single prize?

$$\text{Probability} = \frac{3}{10{,}000}$$

Special Multiplication Rule

#2 $$P(A \text{ and } B) = P(A) \times P(B)$$

[Where the probability of A and B equals the probability of A times the probability of B]

If events are independent ("no influence on one another"), we simply multiply them together.

Example What is the probability of tossing a coin twice and obtaining heads on both the first and second toss?

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

General Multiplication Rule

#3 $$P(A \text{ and } B) = P(A) \times P(B/A)$$

[Where the probability of A and B equals the probability of A times the probability of B, given that A has already occurred]

If events are not independent ("they influence one another"), we must adjust the second event based on its influence from the first event.

Example A bag contains six marbles, three blue and three green. What is the probability of blindly reaching into the bag and pulling out two green marbles?

$$\frac{3}{6} \times \frac{2}{5} = \frac{6}{30} = \frac{1}{5}$$

Special Addition Rule

#4 $P(A \text{ or } B) = P(A) + P(B)$

[Where the probability of A or B equals the probability of A added to the probability of B]

If events are mutually exclusive ("there is no overlap"), then we just add the probability of the events together.

Example The probability that Sam will go to prep school in Switzerland is 50 percent, while the probability that he will go to prep school in England is 25 percent. What is the probability that he will choose to go to prep school in either Switzerland or England?

50% + 25% = 75%

General Addition Rule

#5 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

[Where the probability of A or B equals the probability of A added to the probability of B minus the probability of A and B]

If events are not mutually exclusive ("there is overlap"), then we must subtract out the overlap subsequent to adding the events.

Example The probability that tomorrow will be *rainy* is 30 percent. The probability that tomorrow will be *windy* is 20 percent. What is the probability that tomorrow's weather will be either rainy or windy?

30% + 20% – (30% × 20%)
50% – 6% = 44%

Author's note: Let's quickly contrast what is commonly referred to as the inclusive "or" and the exclusive "or." The problem above, highlighting Probability Rule 5, is governed by an inclusive "or." It is reasonable to assume that tomorrow's weather can be both rainy and windy. The problem is effectively asking, "What is the probability that tomorrow's weather will be either rainy or windy or both rainy and windy." The inclusive "or" occurs whenever there is overlap. The previous problem, which appears in support of Probability Rule 4, effectively asks, "What is the probability that he (Sam) will choose to go to prep school in either Switzerland or England, but not in both countries?" The choice between going to prep school in one of two countries is clearly a mutually exclusive one and we treat that particular problem as involving an exclusive "or."

With regard to the General Addition Rule, the reason that we subtract out the overlap is because we do not want to count it twice. When two events overlap, both events contain that same overlap. Thus, it must be subtracted once in order not to "double" count it.

Complement Rule

#6 $P(A) = 1 - P(not\ A)$

[Where the probability of A equals one minus the probability A not occurring]
The Complement Rule of Probability describes the *subtracting of probabilities* rather than the *adding or multiplying of probabilities*. To calculate the probability of an event using this rule, we ask what is the probability of a given event not occurring and subtract this result from 1.

Example What is the probability of rolling a pair of dice and not rolling double sixes?

$$1 - \frac{1}{36} = \frac{35}{36}$$

In short, this probability equals one minus the probability of rolling double sixes.

Rule of Enumeration

#7 If there are x ways of doing one thing, y ways of doing a second thing, and z ways of doing a third thing, then the number of ways doing all these things is $x \times y \times z$. This is known as the Rule of Enumeration.

Author's note: Technically, the Rule of Enumeration falls under neither the umbrella of "probability," not permutation or combination. But for practical reasons, it is most often discussed along with probability.

Example Fast-Feast Restaurant offers customers a set menu with a choice of one of each of the following: 2 different salads, 3 different soups, 5 different entrees, 3 different desserts, and coffee or tea. How many possibilities are there with respect to how a customer can take his or her dinner?

$2 \times 3 \times 5 \times 3 \times 2 = 180$

Permutations

#8 (i) without replacement $\quad {}_nP_r = \dfrac{n!}{(n-r)!}$

[Where n = total number of items and r = number of items we are taking or arranging]

#9 ${}_nP_n = n!$

[Shortcut formula when all items are taken together]

Example How many ways can a person display (or arrange) four different books on a shelf?

$${}_nP_r = \frac{n!}{(n-r)!}$$

$$ {}_4P_4 = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4!}{1} = 4! = 24 $$

Also, shortcut formula: $n! = 4! = 24$

#10 (ii) with replacement n^r

Example How many four-digit codes can be made from the numbers 1, 2, 3, and 4, if the same numbers can be displayed more than once?

n^r $4^4 = 256$

Author's note: Permutation with replacement (i.e., n^r) technically falls under the Rule of Enumeration. It is included here for ease of presentation. For a problem to be considered a permutation, the permutation formula must be applicable.

Combinations

#11 $$ {}_nC_r = \frac{n!}{r!(n-r)!} $$

[Where n = total number of items taken and r = the number of items we are choosing or selecting]

Example How many ways can a person choose three of four colors for the purpose of painting the inside of a house?

$$ {}_nC_r = \frac{n!}{r!\,(n-r)!} $$

$$ {}_4C_3 = \frac{4!}{3!\,(4-3)!} = \frac{4!}{3! \times 1!} = 4 $$

Additional formulas:

Joint Permutations

#12 $$ {}_nP_r \times {}_nP_r = \frac{n!}{(n-r)!} \times \frac{n!}{(n-r)!} $$

Example A tourist plans to visit three of five Western European cities and then proceed to visit two of four Eastern European cities. At the planning stage, how many itineraries are possible?

$$ {}_5P_3 \times {}_4P_2 $$

$$\frac{5!}{(5-3!)} \times \frac{4!}{(4-2!)}$$

$$\frac{5!}{2!} \times \frac{4!}{2!}$$

$$\frac{5 \times 4 \times 3 \times \cancel{2 \times 1!}}{\cancel{2 \times 1!}} \times \frac{4 \times 3 \times \cancel{2 \times 1!}}{\cancel{2 \times 1!}}$$

$60 \times 12 = 720$

Multiplying outcomes, rather than adding them, is consistent with the treatment afforded the Rule of Enumeration.

Joint Combinations

#13 $${}_nC_r \times {}_nC_r = \frac{n!}{r!(n-r)!} \times \frac{n!}{r!(n-r)!}$$

Example A special marketing task force is to be chosen from five professional golfers and five professional tennis players. If the final task force chosen is to consist of three golfers and three tennis players, then how many different task forces are possible?

$${}_5C_3 \times {}_5C_3$$

$$\frac{5!}{3!(5-3)!} \times \frac{5!}{3!(5-3)!}$$

$$\frac{5!}{3!(2)!} \times \frac{5!}{3!(2)!}$$

$$\frac{5 \times \cancel{4}^{2} \times \cancel{3 \times 2 \times 1}}{\cancel{3 \times 2 \times 1} \times \cancel{2 \times 1}} \times \frac{5 \times \cancel{4}^{2} \times \cancel{3 \times 2 \times 1}}{\cancel{3 \times 2 \times 1} \times \cancel{2 \times 1}}$$

$10 \times 10 = 100$

Repeated Letters or Numbers (Permutations)

#14 $\frac{n!}{x!y!z!}$ where x, y, and z are different but identical letters or numbers.

Example How many four-numeral codes can be created using the four numbers 0, 0, 1, and 2?

$$= \frac{4!}{2!} = \frac{4 \times 3 \times \cancel{2 \times 1}}{\cancel{2 \times 1}} = 12$$

Note that the two zeros are repeated numbers.

MULTIPLE-CHOICE PROBLEMS

Distance-Rate-Time Problems

1. River Boat ( )

A river boat leaves Silver Town and travels upstream to Gold Town at an average speed of 6 kilometers per hour. It returns by the same route at an average speed of 9 kilometers per hour. What is the average speed for the round-trip in kilometers per hour?

A) 7.0
B) 7.1
C) 7.2
D) 7.5
E) 8.0

2. Run-Run ( )

If Susan takes 9 seconds to run y yards, how many *minutes* will it take her to run x yards at the same rate?

A) $\frac{xy}{9}$

B) $\frac{9x}{60y}$

C) $\frac{60xy}{9}$

D) $\frac{xy}{540}$

E) $\frac{540x}{y}$

3. Forgetful Timothy ( )

Timothy leaves home for school, riding his bicycle at a rate of 9 miles per hour. Fifteen minutes after he leaves, his mother sees Timothy's math homework lying on his bed and immediately leaves home to bring it to him. If his mother drives at 36 miles per hour, how far (in terms of miles) must she drive before she reaches Timothy?

A) $\frac{1}{3}$
B) 3
C) 4
D) 9
E) 12

4. P & Q ( )

P and Q are the only two applicants qualified for a short-term research project that pays 600 dollars in total. Candidate P has more experience and, if hired, would be paid 50 percent more per hour than candidate Q would be paid. Candidate Q, if hired, would require 10 hours more than candidate P to do the job. Candidate P's hourly wage is how many dollars greater than candidate Q's hourly wage?

A) $10
B) $15
C) $20
D) $25
E) $30

5. Submarine ( )

On a reconnaissance mission, a state-of-the-art nuclear powered submarine traveled 240 miles to reposition itself in the proximity of an aircraft carrier. This journey would have taken 1 hour less if the submarine had traveled 20 miles per hour faster. What was the average speed, in miles per hour, for the actual journey?

A) 20
B) 40
C) 60
D) 80
E) 100

6. Sixteen-Wheeler ( )

Two heavily loaded sixteen-wheeler transport trucks are 770 kilometers apart, sitting at two rest stops on opposite sides of the same highway. Driver A begins heading down the highway driving at an average speed of 90 kilometers per hour. Exactly one hour later, Driver B starts down the highway toward Driver A, maintaining an average speed of 80 kilometers per hour. How many kilometers *farther* than Driver B, will Driver A have driven when they meet and pass each other on the highway?

A) 90
B) 130
C) 150
D) 320
E) 450

Age Problems

7. Elmer ( )

Elmer, the circus Elephant, is currently three times older than Leo, the circus Lion. In five years from now, Leo the circus Lion will be exactly half as old as Elmer, the circus Elephant. How old is Elmer today?

A) 10
B) 15
C) 17
D) 22
E) 25

Average Problems

8. Three's Company ( )

The average (arithmetic mean) of four numbers is $4x + 3$. If one of the numbers is x, what is the average of the other three numbers?

A) $x + 1$
B) $3x + 3$
C) $5x + 1$
D) $5x + 4$
E) $15x + 12$

9. Fourth Time Lucky ( )

On his first 3 tests, Rajeev received an average score of N points. If on his fourth test, he exceeds his previous average score by 20 points, what is his average score for his first 4 tests?

A) N
B) $N + 4$
C) $N + 5$
D) $N + 10$
E) $N + 20$

10. Vacation ( )

P persons have decided to rent a van to tour while on holidays. The price of the van is x dollars and each person is to pay an equal share. If D persons cancel their trip thus failing to pay their share, which of the following represents the *additional* number of dollars per person that each remaining person must pay in order to still rent the van?

A) Dx

B) $\dfrac{x}{P-D}$

C) $\dfrac{Dx}{P-D}$

D) $\dfrac{Dx}{P(P-D)}$

E) $\dfrac{x}{P(P-D)}$

Work Problems

11. Disappearing Act ( )

Working individually, Deborah can wash all the dishes from her friend's wedding banquet in 5 hours and Tom can wash all the dishes in 6 hours. If Deborah and Tom work together but independently at the task for 2 hours, at which point Tom leaves, how many remaining hours will it take Deborah to complete the task alone?

A) $\dfrac{4}{15}$

B) $\dfrac{3}{11}$

C) $\dfrac{4}{3}$

D) $\dfrac{15}{11}$

E) $\dfrac{11}{2}$

12. Exhibition ( )

If it takes 70 workers 3 hours to disassemble the exhibition rides at a small amusement park, how many hours would it take 30 workers to do this same job?

A) $\frac{40}{3}$

B) 11

C) 7

D) $\frac{7}{3}$

E) $\frac{9}{7}$

13. Legal ( )

A group of 4 junior lawyers require 5 hours to complete a legal research assignment. How many hours would it take a group of three legal assistants to complete the same research assignment assuming that a legal assistant works at two-thirds the rate of a junior lawyer?

A) 13
B) 10
C) 9
D) 6
E) 5

Picture Frame, Rug, or Border Problems

14. Persian Rug ( )

A Persian rug set on a dining room floor measures a inches by b inches, which includes the actual rug design and a solid colored border c inches. Which algebraic expression below represents the *area* of the solid colored border in square inches?

A) $ab - 4c$

B) $a + b - [(a - c) + (b - c)]$

C) $2a + 2b - [2(a - 2c) + 2(b - 2c)]$

D) $ab - (a - c)(b - c)$

E) $ab - (a - 2c)(b - 2c)$

Mixture Problems

15. Nuts ( )

A wholesaler wishes to sell 100 pounds of mixed nuts at \$2.50 a pound. She mixes peanuts worth \$1.50 a pound with cashews worth \$4.00 a pound. How many pounds of cashews must she use?

A) 40
B) 45
C) 50
D) 55
E) 60

16. Gold ( )

An alloy weighing 24 ounces is 70 percent gold. How many ounces of pure gold must be added to create an alloy that is 90 percent gold?

A) 6
B) 9
C) 12
D) 24
E) 48

17. Evaporation ( )

How many liters of water must be evaporated from 50 liters of a 3-percent sugar solution to get a 10-percent solution?

A) 35
B) $33\frac{1}{3}$
C) 27
D) $16\frac{2}{3}$
E) 15

Group Problems

18. Standardized Test ( )

If 85 percent of the test takers taking an old paper and pencil GMAT exam answered the first question on a given math section correctly, and 75 percent of the test takers answered the second question correctly, and 5 percent of the test takers answered neither question correctly, what percent answered *both* correctly?

A) 60%
B) 65%
C) 70%
D) 75%
E) 80%

19. Language Classes ( )

According to the admissions and records office of a major university, the schedules of X first-year college students were inspected and it was found that S number of students were taking a Spanish course, F number of students were taking a French course, and B number of students were taking both a Spanish and a French Course. Which of the following expressions gives the percentage of students whose schedules were inspected who were taking *neither* a Spanish course *nor* a French course?

A) $100 \times \dfrac{X}{B+F+S}$

B) $100 \times \dfrac{B+F+S}{X}$

C) $100 \times \dfrac{X-F-S}{X}$

D) $100 \times \dfrac{X+B-F-S}{X}$

E) $100 \times \dfrac{X-B-F-S}{X}$

20. German Cars ( )

The *New Marketing Journal* conducted a survey of wealthy German car owners. According to the survey, all wealthy car owners owned one or more of the following three brands: BMW, Mercedes, or Porsche. Respondents' answers were grouped as follows: 45 owned BMW cars, 38 owned Mercedes cars, and 27 owned Porsche cars. Of these, 15 owned both BMW and Mercedes cars, 12 owned both Mercedes and Porsche cars, 8 owned both BMW and Porsche cars, and 5 persons owned all three types of cars. How many different individuals were surveyed?

A) 70
B) 75
C) 80
D) 110
E) 130

Matrix Problems

21. Single ( )

In a graduate physics course, 70 percent of the students are male and 30 percent of the students are married. If two-sevenths of the male students are married, what fraction of the female students is single?

A) $\frac{2}{7}$

B) $\frac{1}{3}$

C) $\frac{1}{2}$

D) $\frac{2}{3}$

E) $\frac{5}{7}$

22. Batteries ( )

One-fifth of the batteries produced by an upstart factory are defective and one-quarter of all batteries produced are rejected by the quality control technician. If one-tenth of the non-defective batteries are rejected by mistake, and if all the batteries not rejected are sold, then what percent of the batteries sold by the factory are defective?

A) 4%
B) 5%
C) 6%
D) 8%
E) 12%

23. Experiment ( )

Sixty percent of the rats included in an experiment were female rats. If some of the rats died during an experiment and 70 percent of the rats that died were male rats, what was the ratio of the death rate among the male rats to the death rate among the female rats?

A) 7 : 2
B) 7 : 3
C) 2 : 7
D) 3 : 7
E) Cannot be determined from the information given

Price-Cost-Volume-Profit Problems

24. Garments ( )

If s shirts can be purchased for d dollars, how many shirts can be purchased for t dollars?

A) sdt

B) $\frac{ts}{d}$

C) $\frac{td}{s}$

D) $\frac{d}{st}$

E) $\frac{s}{dt}$

25. Pete's Pet Shop ( )

At Pete's Pet Shop, 35 cups of bird seed are used every 7 days to feed 15 parakeets. How many cups of bird seed would be required to feed 9 parakeets for 12 days?

A) 32
B) 36
C) 39
D) 42
E) 45

26. Sabrina ( )

Sabrina is contemplating a job switch. She is thinking of leaving her job paying $85,000 per year to accept a sales job paying $45,000 per year plus 15 percent commission for each sale made. If each of her sales is for $1,500, what is the least number of sales she must make per year if she is not to lose money because of the job change?

A) 57
B) 177
C) 178
D) 377
E) 378

27. Delicatessen ( )

A large delicatessen purchased p pounds of cheese for c dollars per pound. If d pounds of the cheese had to be discarded due to spoilage and the delicatessen sold the rest for s dollars per pound, which of the following represents the gross profit on the sale of the purchase? (*gross profit* equals *sales revenue* minus *product cost*)

A) $(p - d)(s - c)$
B) $s(p - d) - pc$
C) $c(p - d) - ds$
D) $d(s - c) - pc$
E) $pc - ds$

28. Prototype ( )

A Prototype fuel-efficient car (P-Car) is estimated to get 80% more miles per gallon of gasoline than does a traditional fuel-efficient car (T-Car). However, the P-Car requires a special type of gasoline that costs 20% more per gallon than does the gasoline used by a T-Car. If the two cars are driven the same distance, what percent less than the money spent on gasoline for the T-Car is the money spent on gasoline for the P-Car?

A) $16\frac{2}{3}\%$

B) $33\frac{1}{3}$

C) 50%

D) 60%

E) $66\frac{2}{3}\%$

Least-Common-Multiple Word Problems

29. Lights ( )

The Royal Hawaiian Hotel decorates its Rainbow Christmas Tree with non-flashing white lights and a series of colored flashing lights—red, blue, green, orange, and yellow. The red lights turn red every 20 seconds, the blue lights turn blue every 30 seconds, the green lights turn green every 45 seconds, the orange lights turn orange every 60 seconds, and yellow lights turn yellow every 1 minute and 20 seconds. The manager plugs the tree in for the first time on December 1st precisely at midnight and all lights begin their cycle at exactly the same time. If the five colored lights flash simultaneously at midnight, what is the next time all five colored lights will all flash together at the exact same time?

A) 0:03 AM
B) 0:04 AM
C) 0:06 AM
D) 0:12 AM
E) 0:24 AM

General Algebraic Word Problems

30. Hardware ( )

Hammers and wrenches are manufactured at a uniform weight per hammer and a uniform weight per wrench. If the total weight of two hammers and three wrenches is one-third that of 8 hammers and 5 wrenches, then the total weight of one wrench is how many times that of one hammer?

A) $\frac{1}{2}$

B) $\frac{2}{3}$

C) 1

D) $\frac{3}{2}$

E) 2

31. Snooker ( )

A snooker tournament charges $45.00 for VIP seats and $15.00 for general admission ("regular" seats). On a certain night, a total of 320 tickets were sold, for a total cost of $7,500. How many fewer tickets were sold that night for VIP seats than for general admission seats?

A) 70
B) 90
C) 140
D) 230
E) 250

32. Chili Paste ( )

Each week a restaurant serving Mexican food uses the same volume of chili paste, which comes in either 25-ounce cans or 15-ounce cans of chili paste. If the restaurant must order 40 more of the smaller cans than the larger cans to fulfill its weekly needs, then how many *smaller* cans are required to fulfill its weekly needs?

A) 60
B) 70
C) 80
D) 100
E) 120

33. Premium ()

The price of 5 kilograms of premium fertilizer is the same as the price of 6 kilograms of regular fertilizer. If the price of premium fertilizer is y cents per kilogram more than the price of regular fertilizer, what is the price, in cents, per kilogram of premium fertilizer?

A) $\frac{y}{30}$
B) $\frac{5}{6}y$
C) $\frac{6}{5}y$
D) $5y$
E) $6y$

Function Problems

34. Function ()

If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{x^2 + 7}$, what is the value of $f(g(3))$?

A) 1
B) 2
C) 3
D) 4
E) 5

Algebraic Fractions

35. Rescue ( )

If $a = \dfrac{b-d}{c-d}$, then $d =$

A) $\dfrac{b+a}{c+a}$

B) $\dfrac{b-a}{c-a}$

C) $\dfrac{bc-a}{bc+a}$

D) $\dfrac{b-ac}{1-a}$

E) $\dfrac{b-ac}{a-1}$

36. Hodgepodge ( )

The expression $\dfrac{\frac{1}{h}}{1-\frac{1}{h}}$, where h is not equal to 0 and 1, is equivalent to which of the following?

A) $1-h$

B) $h-1$

C) $\dfrac{1}{h-1}$

D) $\dfrac{1}{1-h}$

E) $\dfrac{h}{h-1}$

Fractions and Decimals

37. Mirage ( )

Which of the following has the greatest value?

A) $\frac{10}{11}$

B) $\frac{4}{5}$

C) $\frac{7}{8}$

D) $\frac{21}{22}$

E) $\frac{5}{6}$

38. Deceptive ( )

Dividing 100 by 0.75 will lead to the same mathematical result as multiplying 100 by which number?

A) 0.25
B) 0.75
C) 1.25
D) 1.33
E) 1.75

39. Spiral ( )

In a certain sequence, the first term is 2, and each successive term is 1 more than the reciprocal of the term that immediately precedes it. What is the fifth term in this sequence?

A) $\frac{13}{8}$

B) $\frac{21}{13}$

C) $\frac{8}{5}$

D) $\frac{5}{8}$

E) $\frac{8}{13}$

Percentage Problems

40. Discount ( )

A discount of 10 percent on an order of goods followed by a discount of 30 percent amounts to

A) the same as one 13 percent discount
B) the same as one 27 percent discount
C) the same as one 33 percent discount
D) the same as one 37 percent discount
E) the same as one 40 percent discount

41. Inflation ( )

An inflationary increase of 20 percent on an order of raw materials followed by an inflationary increase of 10 percent amounts to

A) the same as one 22 percent inflationary increase
B) the same as one 30 percent inflationary increase
C) the same as an inflationary increase of 10 percent followed by an inflationary increase of 20 percent
D) less than an inflationary increase of 10 percent followed by an inflationary increase of 20 percent
E) more than an inflationary increase of 10 percent followed by an inflationary increase of 20 percent

42. Gardener ( )

A gardener increased the length of his rectangle-shaped garden by 40 percent and decreased its width by 20 percent. The area of the new garden

A) has increased by 20 percent
B) has increased by 12 percent
C) has increased by 8 percent
D) is exactly the same as the old area
E) cannot be expressed in percentage terms without actual numbers

43. Microbrewery ( )

Over the course of a year, a certain microbrewery increased its beer output by 70 percent. At the same time, it decreased its total working hours by 20 percent. By what percent did this factory increase its output per hour?

A) 50%
B) 90%
C) 112.5%
D) 210%
E) 212.5%

44. Squaring Off ( )

If the sides of a square are doubled in length, the area of the original square is now how many times as large as the area of the resultant square?

A) 25%
B) 50%
C) 100%
D) 200%
E) 400%

45. Diners ( )

A couple spent \$264 in total while dining out and paid this amount using a credit card. The \$264 figure included a 20 percent tip which was paid on top of the price of the food which already included a sales tax of 10 percent. What was the actual price of the meal before tax and tip?

A) \$184
B) \$200
C) \$204
D) \$216
E) \$232

46. Investments ( )

A lady sold two small investment properties, A and B, for $24,000 each. If she sold property A for 20 percent more than she paid for it, and sold property B for 20 percent less than she paid for it, then, in terms of the net financial effect of these two investments (excluding taxes and expenses), we can conclude that the lady

A) broke even
B) had an overall gain of $1,200
C) had an overall loss of $1,200
D) had an overall gain of $2,000
E) had an overall loss of $2,000

Ratios and Proportions

47. Earth Speed ( )

The Earth travels around the Sun at an approximate speed of 20 miles per second. This speed is how many kilometers per hour? [1km = 0.6 miles]

A) 2,000
B) 12,000
C) 43,200
D) 72,000
E) 120,000

48. Rum & Coke ( )

A drink holding 6 ounces of an alcoholic drink that is 1 part rum to 2 parts coke is added to a jug holding 32 ounces of an alcoholic drink that is 1 part rum to 3 parts coke. What is the ratio of rum to coke in the resulting mixture?

A) 2 : 5
B) 5 : 14
C) 3 : 5
D) 4 : 7
E) 14 : 5

49. Millionaire ( )

For every \$20 that a billionaire spends, a millionaire spends the equivalent of 20 cents. For every \$4 that a millionaire spends, a yuppie spends the equivalent of \$1. The ratio of money spent by a yuppie, millionaire, and billionaire can be expressed as

A) 1 : 4 : 400
B) 1 : 4 : 100
C) 20 : 4 : 1
D) 100 : 4 : 1
E) 400 : 4 : 1

50. Deluxe ( )

At Deluxe paint store, Fuchsia paint is made by mixing 5 parts of red paint with 3 parts of blue paint. Mauve paint is made by mixing 3 parts of red paint with 5 parts blue paint. How many liters of blue paint must be added to 24 liters of Fuchsia to change it to Mauve paint?

A) 9
B) 12
C) 15
D) 16
E) 18

51. Rare Coins ( )

In a rare coin collection, all coins are either pure gold or pure silver, and there is initially one gold coin for every three silver coins. With the addition of 10 more gold coins to the collection, the ratio of gold coins to silver coins is 1 to 2. Based on this information, how many total coins are there now in this collection (after the acquisition)?

A) 40
B) 50
C) 60
D) 80
E) 90

52. Coins Revisited ( )

In a rare coin collection, one in six coins is gold, and all coins are either gold or silver. If 10 silver coins were to be subsequently traded for an additional 10 gold coins, the ratio of gold coins to silver coins would be 1 to 4. Based on this information, how many gold coins would there be in this collection after the proposed trade?

A) 50
B) 60
C) 180
D) 200
E) 300

Squares and Cubes

53. Plus-Zero ( )

If $x > 0$, which of the following could be true?

I. $x^3 > x^2$

II. $x^2 = x$

III. $x^2 > x^3$

A) I only
B) I & II
C) II & III
D) All of the above
E) None of the above

54. Sub-Zero ( )

If $x < 0$, which of the following must be true?

I. $x^2 > 0$

II. $x - 2x > 0$

III. $x^3 + x^2 < 0$

A) I only
B) I & II
C) II & III
D) All of the above
E) None of the above

Exponent Problems

55. Solar Power ( )

The mass of the sun is approximately 2×10^{30} kg and the mass of the moon is approximately 8×10^{12} kg. The mass of the sun is approximately how many times the mass of the moon?

A) 4.0×10^{-18}

B) 2.5×10^{17}

C) 4.0×10^{18}

D) 2.5×10^{19}

E) 4.0×10^{42}

56. Bacteria ( )

A certain population of bacteria doubles every 10 minutes. If the number of bacteria in the population initially was 10^5, then what was the number in the population 1 hour later?

A) $2(10^5)$

B) $6(10^5)$

C) $(2^6)(10^5)$

D) $(10^6)(10^5)$

E) $(10^5)^6$

57. K.I.S.S. ( )

If a is a positive integer, then $3^a + 3^{a+1} =$

A) 4^a

B) $3^a - 1$

C) $3^{2a} + 1$

D) $3^a(a-1)$

E) $4(3^a)$

58. Triplets ()

$3^{10} + 3^{10} + 3^{10} =$

A) 3^{11}

B) 3^{13}

C) 3^{30}

D) 9^{10}

E) 9^{30}

59. The Power of 5 ()

If $5^5 \times 5^7 = (125)^x$, then what is the value of x?

A) 2
B) 3
C) 4
D) 5
E) 6

60. M&N ()

If $m > 1$ and $n = 2^{m-1}$, then $4^m =$

A) $16n^2$

B) $4n^2$

C) n^2

D) $\dfrac{n^2}{4}$

E) $\dfrac{n^2}{16}$

61. Incognito ( )

Which of the following fractions has the greatest value?

A) $\frac{25}{(2^4)(3^3)}$

B) $\frac{5}{(2^2)(3^3)}$

C) $\frac{4}{(2^3)(3^2)}$

D) $\frac{36}{(2^3)(3^4)}$

E) $\frac{76}{(2^4)(3^4)}$

62. Chain Reaction ( )

If $x - \frac{1}{2^6} - \frac{1}{2^7} - \frac{1}{2^8} = \frac{2}{2^9}$, then $x =$

A) $\frac{1}{2}$

B) $\frac{1}{2^3}$

C) $\frac{1}{2^4}$

D) $\frac{1}{2^5}$

E) $\frac{1}{2^9}$

Radical Problems

63. Simplify ( )

$$\sqrt{\frac{12\times3+4\times16}{6}}=$$

A) $\frac{5\sqrt{6}}{3}$

B) $\sqrt{22}$

C) $\sqrt{6}+4$

D) $\frac{8\sqrt{15}}{3}$

E) $16\frac{2}{3}$

64. Tenfold ( )

$$\frac{\sqrt{10}}{\sqrt{0.001}}=$$

A) 10,000
B) 1,000
C) 100
D) 1
E) Can be expressed only as a non-integer

65. Strange ( )

The expression $\frac{1-\sqrt{2}}{1+\sqrt{2}}$ is equivalent to which of the following?

A) $-3+2\sqrt{2}$

B) $1-\frac{2}{3}\sqrt{2}$

C) 0

D) $1+\frac{2}{3}\sqrt{2}$

E) $3+2\sqrt{2}$

Inequality Problems

66. Two-Way Split ( )

If $-x^2+16<0$, which of the following must be true?

A) $-4>x>4$
B) $-4<x>4$
C) $-4<x<4$
D) $-4\le x\ge 4$
E) $-4\ge x\ge 4$

Prime Number Problems

67. Primed ( )

The "primeness" of a positive integer x is defined as the positive difference between its largest and smallest prime factors. Which of the following has the greatest primeness?

A) 10
B) 12
C) 14
D) 15
E) 18

68. Odd Man Out ()

If P represents the product of the first 13 positive integers, which of the following must be true?

I. P is an odd number
II. P is a multiple of 17
III. P is a multiple of 24

A) I only
B) II only
C) III only
D) None of the above
E) All of the above

Remainder Problems

69. Remainder ()

When the integer k is divided by 7, the remainder is 5. Which of the following expressions below when divided by 7, will have a remainder of 6?

I. $4k + 7$
II. $6k + 4$
III. $8k + 1$

A) I only
B) II only
C) III only
D) I and II only
E) I, II and III

70. Double Digits ()

How many two-digit whole numbers yield a remainder of 3 when divided by 10 and also yield a remainder of 3 when divided by 4?

A) One
B) Two
C) Three
D) Four
E) Five

Symbolism Problems

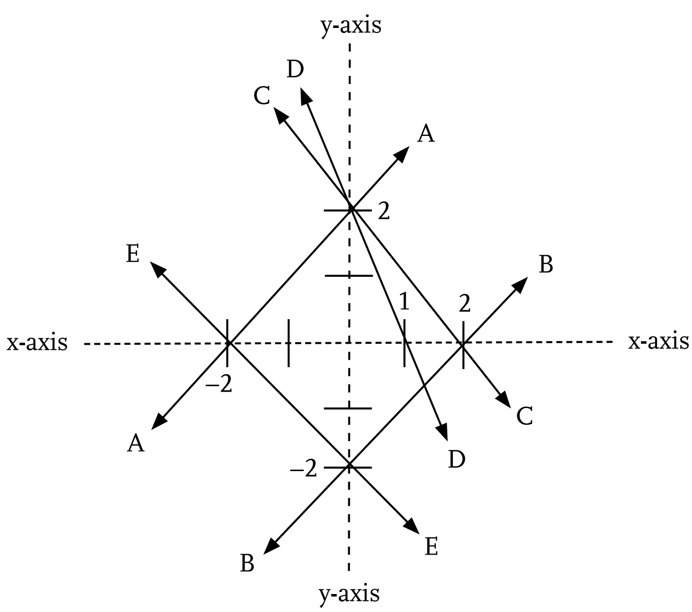
71. Visualize ( )

For all real numbers V, the operation V^* is defined by the equation $V^* = V - \frac{V}{2}$. If $(V^*)^* = 3$, then $V =$

A) 12
B) 6
C) 4
D) $\sqrt{12}$
E) -12

Coordinate Geometry Problems

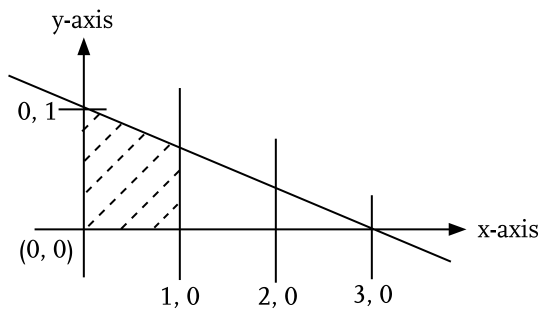
72. Masquerade ( )



Which of the above lines fit the equation $y = -2x + 2$?

A) Line A
B) Line B
C) Line C
D) Line D
E) Line E

73. Boxed In ( )



In the rectangular coordinate system above, the shaded region is bounded by a straight line. Which of the following is NOT an equation of one of the boundary lines?

A) $x = 0$

B) $y = 0$

C) $x = 1$

D) $x - 3y = 0$

E) $y + \frac{1}{3}x = 1$

74. Intercept ( )

In the rectangular coordinate system, what is the *x*-intercept of a line passing through (10,3) and (–6, –5)?

A) 4
B) 2
C) 0
D) –2
E) –4

Plane Geometry Problems

75. Magic ( )

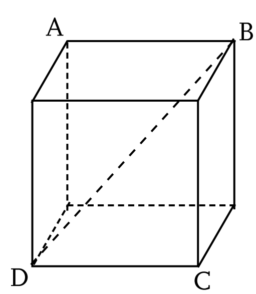
What is the ratio of the circumference of a circle to its diameter?

A) π
B) 2π
C) π^2
D) $2\pi r$
E) varies depending on the size of the circle

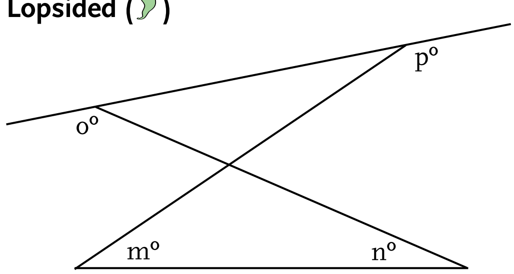
76. Kitty Corner ( )

The figure below is a cube with each side equal to 2 units. What is the length (in units) of diagonal BD? (Note: BD is drawn diagonally from bottom left-hand corner in the front to top right-hand corner at the back.)

A) $2\sqrt{2}$
B) $2\sqrt{3}$
C) $3\sqrt{2}$
D) $3\sqrt{3}$
E) $4\sqrt{3}$



77. Lopsided ( )



Note: Figure not drawn to scale

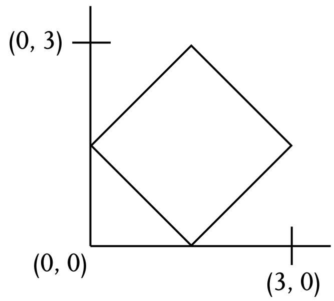
In the figure above, $m + n = 110$. What is the value of $o + p$?

A) 70
B) 110
C) 250
D) 270
E) 330

78. Diamond ( )

The figure below is a square. What is its perimeter (measured in units)?

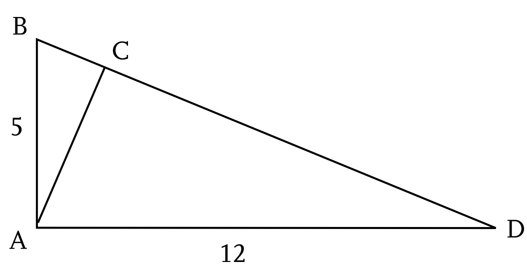
A) $6\sqrt{2}$
B) 9
C) 12
D) $12\sqrt{2}$
E) 18



79. AC ( )

In the right triangle ABD below, AC is perpendicular to BD. If $AB = 5$ and $AD = 12$, then AC is equal to?

A) $\frac{30}{13}$
B) $\sqrt{12}$
C) 4
D) $2\sqrt{5}$
E) $\frac{60}{13}$



80. Circuit ( )

A rectangular circuit board is designed to have a width of W inches, a length of L inches, a perimeter of P inches, and an area of A square inches. Which of the following equations must be true?

A) $2W^2 + PW + 2A = 0$
B) $2W^2 - PW + 2A = 0$
C) $2W^2 - PW - 2A = 0$
D) $W^2 + PW + A = 0$
E) $W^2 - PW + 2A = 0$

81. Victorian ( )

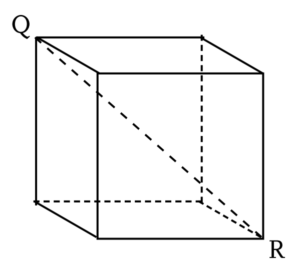
A professional painter is painting the window frames of an old Victorian House. The worker has a ladder that is exactly 25 feet in length which he will use to paint two sets of window frames. To reach the first window frame, he places the ladder so that it rests against the side of the house at a point exactly 15 feet above the ground. When he finishes, he proceeds to reposition the ladder to reach the second window so that now the ladder rests against the side of the house at a point exactly 24 feet above the ground. How much *closer* to the base of the house has the bottom of the ladder now been moved?

A) 7
B) 9
C) 10
D) 13
E) 27

82. QR ( )

If segment QR in the cube below has length $4\sqrt{3}$ inches, what is the volume of the cube (in cubic inches)?

A) 16
B) 27
C) 64
D) 81
E) 125



83. Cornered ( )

Viewed from the outside inward, the figure below depicts a square-circle-square-circle, each enclosed within the other. If the area of square ABCD is 2 square units, then which of the following expresses the area of the darkened corners of square EFGH?

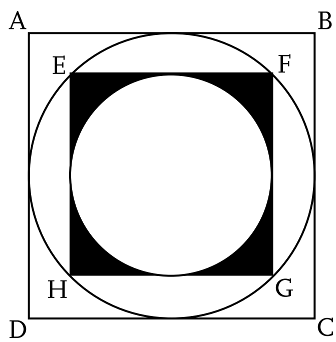
A) $2-\frac{1}{4}\pi$

B) $2-\frac{1}{2}\pi$

C) $1-\frac{1}{4}\pi$

D) $\frac{1}{2}-\frac{1}{8}\pi$

E) $1-\frac{1}{2}\pi$



84. Woozy ( )

In the equilateral triangle below, each side has a length of 4 units. If PQ has a length of 1 unit and TQ is perpendicular to PR, what is the area of region QRST?

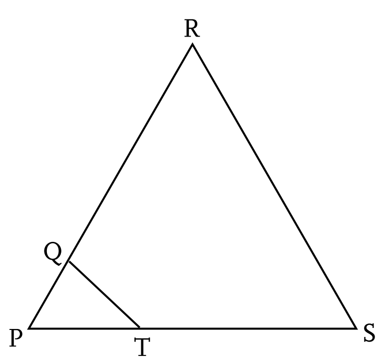
A) $\frac{1}{3}\sqrt{3}$

B) $3\sqrt{3}$

C) $\frac{7}{2}\sqrt{3}$

D) $4\sqrt{3}$

E) $15\sqrt{3}$



Solid Geometry Problems

85. Sphere ( )

A sphere has a radius of x units. If the length of this radius is doubled, then how many times larger, in terms of volume, is the resultant sphere as compared with the original sphere?

A) 1
B) 2
C) 4
D) 8
E) 16

Probability Problems

86. Exam Time ( )

A student is to take her final exams in two subjects. The probability that she will pass the first subject is $\frac{3}{4}$ and the probability that she will pass the second subject is $\frac{2}{3}$. What is the probability that she will pass one exam or the other exam?

A) $\frac{5}{12}$

B) $\frac{1}{2}$

C) $\frac{7}{12}$

D) $\frac{5}{7}$

E) $\frac{11}{12}$

87. Orange & Blue ( )

There are 5 marbles in a bag—2 are orange and 3 are blue. If two marbles are pulled from the bag, what is the probability that at least one will be orange?

A) $\frac{7}{10}$

B) $\frac{3}{5}$

C) $\frac{2}{5}$

D) $\frac{3}{10}$

E) $\frac{1}{10}$

88. Antidote ( )

Medical analysts predict that one-third of all people who are infected by a certain biological agent could be expected to be killed for each day that passes during which they have not received an antidote. What fraction of a group of 1,000 people could be expected to be killed if infected and not treated for three full days?

A) $\frac{16}{81}$

B) $\frac{8}{27}$

C) $\frac{2}{3}$

D) $\frac{19}{27}$

E) $\frac{65}{81}$

89. Sixth Sense ( )

What is the probability of rolling two normal six-sided dice and getting exactly one six?

A) $\frac{1}{36}$

B) $\frac{1}{6}$

C) $\frac{5}{18}$

D) $\frac{11}{36}$

E) $\frac{1}{3}$

90. At Least One ( )

A student is to take her final exams in three subjects. The probability that she will pass the first subject is $\frac{3}{4}$, the probability that she will pass the second subject is $\frac{2}{3}$, and the probability that she will pass the third subject is $\frac{1}{2}$. What is the probability that she will pass at least one of these three exams?

A) $\frac{1}{4}$

B) $\frac{11}{24}$

C) $\frac{17}{24}$

D) $\frac{3}{4}$

E) $\frac{23}{24}$

91. Coin Toss ( )

What is the probability of tossing a coin five times and having heads appear at most three times?

A) $\frac{1}{16}$

B) $\frac{5}{16}$

C) $\frac{2}{5}$

D) $\frac{13}{16}$

E) $\frac{27}{32}$

Enumeration Problems

92. Hiring ( )

A company seeks to hire a sales manager, a shipping clerk, and a receptionist. The company has narrowed its candidate search and plans to interview all remaining candidates including 7 persons for the position of sales manager, 4 persons for the position of shipping clerk, and 10 persons for the position of receptionist. How many different hirings of these three people are possible?

A) $7 + 4 + 10$

B) $7 \times 4 \times 10$

C) $21 \times 20 \times 19$

D) $7! + 4! + 10!$

E) $7! \times 4! \times 10!$

Permutation Problems

93. Fencing ( )

Four contestants representing four different countries advance to the finals of a fencing championship. Assuming all competitors have an equal chance of winning, how many possibilities are there with respect to how a first-place and second-place medal can be awarded?

A) 6
B) 7
C) 12
D) 16
E) 24

94. Alternating ( )

Six students—3 boys and 3 girls—are to sit side by side for a makeup exam. How many ways could they arrange themselves given that no two boys and no two girls can sit next to one another?

A) 12
B) 36
C) 72
D) 240
E) 720

95. Banana ( )

Which of the following leads to the correct mathematical solution for the number of ways that the letters of the word BANANA could be arranged to create a six-letter code?

A) $6!$

B) $6! - (3! + 2!)$

C) $6! - (3! \times 2!)$

D) $\dfrac{6!}{3! + 2!}$

E) $\dfrac{6!}{3! \times 2!}$

96. Table ( )

How many ways could three people sit at a table with five seats in which two of the five seats will remain empty?

A) 8
B) 12
C) 60
D) 118
E) 120

Combination Problems

97. Singer ( )

For an upcoming charity event, a male vocalist has agreed to sing 4 out of 6 "old songs" and 2 out of 5 "new songs." How many ways can the singer make his selection?

A) 25
B) 50
C) 150
D) 480
E) 600

98. Outcomes ( )

Given that ${}_nP_r = \frac{n!}{(n-r)!}$ and ${}_nC_r = \frac{n!}{r!(n-r)!}$, where n is the total number of items and r is the number of items taken or chosen, which of the following statements are *true* in terms of the number of outcomes generated?

I. ${}_5P_3 > {}_5P_2$

II. ${}_5C_3 > {}_5C_2$

III. ${}_5C_2 > {}_5P_2$

A) I only
B) I & II only
C) I & III only
D) II & III only
E) I, II & III

99. Reunion ( )

If 11 persons meet at a reunion and each person shakes hands exactly once with each of the others, what is the total number of handshakes?

A) $11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
B) $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
C) 11×10
D) 55
E) 45

100. Display ( )

A computer wholesaler sells eight different computers and each is priced differently. If the wholesaler chooses three computers for display at a trade show, what is the probability (all things being equal) that the two most expensive computers will be among the three chosen for display?

A) $\frac{15}{56}$
B) $\frac{3}{28}$
C) $\frac{1}{28}$
D) $\frac{1}{56}$
E) $\frac{1}{168}$

ANSWERS AND EXPLANATIONS

Answers to the pop quiz

Review – Fractions to Percents

$\frac{1}{3}=33\frac{1}{3}\%$ $\frac{2}{3}=66\frac{2}{3}\%$ $\frac{1}{6}=16\frac{2}{3}\%$ $\frac{5}{6}=83\frac{1}{3}\%$

$\frac{1}{8}=12.5\%$ $\frac{3}{8}=37.5\%$ $\frac{5}{8}=62.5\%$ $\frac{7}{8}=87.5\%$

$\frac{1}{9}=11.11\%$ $\frac{5}{9}=55.55\%$

Review – Decimals to Fractions

The answer to the first question is found by adding 1 to the fractional equivalent of 0.2.

$$0.2=\frac{1}{5} \quad \text{Thus, } 1.2=1+\frac{1}{5}=1\frac{1}{5}=\frac{6}{5}$$

The answer to the second question is found by adding 1 to the fractional equivalent of 0.25.

$$0.25=\frac{1}{4} \quad \text{Thus, } 1.25=1+\frac{1}{4}=1\frac{1}{4}=\frac{5}{4}$$

The answer to the third question is found by adding 1 to the fractional equivalent of 0.33.

$$0.33=\frac{1}{3} \quad \text{Thus, } 1.33=1+\frac{1}{3}=1\frac{1}{3}=\frac{4}{3}$$

Each of the following three "fraction" problems equals 1!

$$\frac{5}{4}\times\frac{5}{10}\times\frac{8}{10}\times\frac{2}{1}=\frac{400}{400}=1$$

$$\frac{\frac{3}{4}\times\frac{5}{6}}{\frac{5}{8}}=\frac{\frac{15}{24}}{\frac{5}{8}}=\frac{15}{24}\times\frac{8}{5}=\frac{120}{120}=1$$

$$\frac{\frac{2}{9}}{\frac{1}{3}\times\frac{2}{3}}=\frac{\frac{2}{9}}{\frac{2}{9}}=\frac{2}{9}\times\frac{9}{2}=\frac{18}{18}=1$$

Review – Common Squares from 13 to 30

Fill in the missing numbers to complete the Pythagorean triplets below. Refer to problem 81 (page 75) for more on Pythagorean triplets.

3 : 4 : 5 5 : 12 : 13 7 : 24 : 25 8 : 15 : 17

Review – Common Square Roots

The answer to this square root problem is III, II & I. These statements are, in fact, in order from smallest to largest value, but the question asks to order the values from smallest to largest.

I. $1 + 2.2 = 3.2$
II. $2 + 1.7 = 3.7$
III. $3 + 1.4 = 4.4$

Review – Exponents and Radicals

The order of values, from largest to smallest, is as follows: II, I, III, VI, IV, and V. In terms of actual values, here is a listing:

I. 4
II. $4^2 = 16$
III. $\sqrt{4} = 2$
IV. $\frac{1}{4}$
V. $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$
VI. $\sqrt{\frac{1}{4}} = \frac{1}{2}$

1. River Boat ( )

Choice C
Classification: Distance-Rate-Time Problem
Snapshot: The easiest way to solve this problem is to supply a hypothetical distance over which the riverboat travels. It may therefore be referred to as a "hypothetical distance D-R-T problem."

To find a hypothetical distance over which the riverboat travels, take the Lowest Common Multiple (L-C-M) of 6 and 9. In other words, assume that the distance is 18 kilometers each way. If the boat travels upstream ("going") at 6 kilometers per hour, it will take 3 hours to complete its 18-kilometer journey. If the boat travels downstream ("returning") at 9 kilometers per hour, it will take 2 hours to complete its 18-kilometer journey.

$$R = \frac{D}{T} = \frac{18+18}{2+3} = \frac{36}{5} = 7.2 \text{ kilometers per hour}$$

The trap answer (choice D) is 7.5 kilometers per hour, which is derived from simply averaging 9 kilometers per hour and 6 kilometers per hour. That is:

$$R = \frac{D}{T} = \frac{9+6}{2} = \frac{15}{2} = 7.5 \text{ kilometers per hour}$$

Author's note: For the record, the *algebraic method* used to solve this problem is as follows:

$$R = \frac{D_1 + D_2}{T_1 + T_2}$$

$$R = \frac{x+x}{\frac{x}{6}+\frac{x}{9}} = \frac{2x}{\frac{3x+2x}{18}} = \frac{2x}{\frac{5x}{18}} = \frac{36x}{5x} = \frac{36}{5} = 7.2 \text{ kilometers per hour}$$

2. Run-Run ( )

Choice B
Classification: Distance-Rate-Time Problem
Snapshot: This D-R-T problem expresses its answer in terms of an algebraic expression. Often such problems also require time conversions.

The problem is asking for time, where $T = \frac{D}{R}$. We have algebraic expressions for both speed and distance. Speed is $\frac{y}{9}$ seconds (not 9 seconds over y yards) and distance is x yards.

Therefore: $T = \frac{D}{R}$

$$T = \frac{x \text{ yards}}{\frac{y \text{ yards}}{9 \text{ sec}}} = x \cancel{\text{ yards}} \times \frac{9 \text{ seconds}}{y \cancel{\text{ yards}}} = \frac{9x \text{ seconds}}{y}$$

$$T = \frac{9x \cancel{\text{ seconds}}}{y} \times \frac{1 \text{ min}}{60 \cancel{\text{ seconds}}} = \frac{9x}{60y} \text{ minutes}$$

Author's note: When solving for distance or rate, we *multiply* by 60 when converting from minutes to hours (or seconds to minutes) and *divide* by 60 when converting from hours to minutes (or minutes to seconds). When solving for time, we *divide* by 60 when converting from minutes to hours (or seconds to minutes) and *multiply* by 60 when converting from hours to minutes (or minutes to seconds).

Additional Examples:

i) Solving for distance

Example At the rate of d miles per q minutes, how many miles does a bullet train travel in x hours?

$$D = R \times T$$

$$D = \frac{d \text{ miles}}{q \cancel{\text{ minutes}}} \times x \cancel{\text{ hours}} \times \frac{60 \cancel{\text{ minutes}}}{1 \cancel{\text{ hour}}} = \frac{60dx}{q} \text{ miles}$$

ii) Solving for rate (or speed)

Example A bullet train completes a journey of d miles. If the journey took q minutes, what was the train's speed in miles per hour?

$$R = \frac{D}{T}$$

$$R = \frac{d \text{ miles}}{q \cancel{\text{ minutes}}} \times \frac{60 \cancel{\text{ minutes}}}{1 \text{ hour}} = \frac{60d}{q} \text{ miles per hour}$$

iii) Solving for time

Example Another bullet train completes a journey of d miles. If this train traveled at a rate of z miles per minute, how many hours did the journey take?

$$T = \frac{D}{R}$$

$$T = \frac{d \cancel{\text{ miles}}}{z \cancel{\text{ miles}} / \cancel{\text{minute}}} \times \frac{1 \text{ hour}}{60 \cancel{\text{ minutes}}} = \frac{d}{60z} \text{ hours}$$

3. Forgetful Timothy ( )

Choice B
Classification: Distance-Rate-Time Problem
Snapshot: *Forgetful Timothy* may be called a "catch up D-R-T problem." A slower individual (or machine) starts first and a faster second person (or machine) must catch up.

Like so many difficult D-R-T problems, the key is to view distance as a constant. In other words, the formulas become: $D_1 = R_1 \times T_1$ and $D_2 = R_2 \times T_2$ where $D_1 = D_2$. The key, therefore, is to set the two formulas equal to one another such that $R_1 \times T_1 = R_2 \times T_2$.

	Rate		Time		Distance
Timothy	9	×	T	=	$9T$
Mother	36	×	$T - \frac{1}{4}$	=	$36\,(T - \frac{1}{4})$

Note: Since time here is measured in hours, 15 minutes should be translated as $\frac{1}{4}$ hour.

First, we solve for T:

$$D_{1\ \text{Timothy}} = D_{2\ \text{Mother}}$$

$$(R_1 \times T_1)_{\text{Timothy}} = (R_2 \times T_2)_{\text{Mother}}$$

$$9T = 36(T - \tfrac{1}{4})$$

$$9T = 36T - 9$$

$$9T - 36T = -9$$
$$-27T = -9$$

$$T = \frac{-9}{-27}$$

$$T = \tfrac{1}{3} \text{ hour}$$

Second, we solve for D:
So, if Timothy rode for $\frac{1}{3}$ hour at 9 m.p.h., the distance he covered was 3 miles. It is also true that his mother drove for 3 miles:

$$D = 36 \text{ m.p.h.} \times (\tfrac{1}{3} \text{ hrs} - \tfrac{1}{4} \text{ hrs})$$

$$D = 36 \text{ m.p.h.} \times \tfrac{1}{12} \text{ hrs} = 3 \text{ miles}$$

4. P & Q ( )

Choice A
Classification: Distance-Rate-Time Problem
Snapshot: In this D-R-T problem, output is a constant, although the rates and times of two working individuals differ and must be expressed relative to each other.

The formula, $D_1 = R_1 \times T_1$, links candidate P and candidate Q in so far as distance or output is a constant (i.e., in this case, "total pay" is $600). Set $P = R_1 \times T_1$ and $Q = R_2 \times T_2$ If the work rate of candidate Q is 100 percent or 1.0, then the work rate of candidate P is 150% or 1.5. If candidate P takes T hours, then candidate Q takes $T + 10$ hours.

$$D_{1\ \text{Candidate P}} = D_{2\ \text{Candidate Q}}$$
$$(R_1 \times T_1)_{\text{Candidate P}} = (R_2 \times T_2)_{\text{Candidate Q}}$$
$$1.5(T) = 1.0(T + 10)$$
$$1.5T = T + 10$$
$$0.5T = 10$$
$$T = \frac{10}{0.5}$$
$$T = 20 \text{ hours}$$

Candidate P takes 20 hours (i.e., 10 + 10). Thus candidate P's hourly rate is: $600 ÷ $20 hours = $30 per hour. Candidate Q's time in hours to complete the research equals $T + 10$ or 20 + 10 = 30 hours. Thus, candidate Q's hourly rate is, $600 ÷ 30 hours = $20 per hour. Therefore, since candidate P earns $30 per hour and candidate Q earns $20 per hour, candidate P earns $10 more dollars per hour than candidate Q does.

5. Submarine ( )

Choice C
Classification: Distance-Rate-Time Problem
Snapshot: *Submarine* is a complicated word problem and one which involves factoring. Again, the key is to view "distance" as a constant where $D_1 = D_2$. The key, therefore, is to set the two formulas equal to one another such that $R_1 \times T_1 = R_2 \times T_2$.

	Rate		Time		Distance
Actual	R	×	T	=	240
Hypothetical	$R + 20$	×	$T - 1$	=	240

We now have two distinct equations:

i) $R \times T = 240$
ii) $(R + 20)(T - 1) = 240$

We need to substitute for one of the variables (i.e., R or T) in order the solve for the remaining variable. Practically, we want to find R, so we solve for R in the second equation by first substituting for T in the second equation.

To do this, we solve for T in the first equation ($T = \frac{240}{R}$) in order to substitute for T in the second equation:

$$(R+20)\left(\frac{240}{R} - 1\right) = 240$$

$$240 - R + \frac{20(240)}{R} - 20 = 240$$

Next multiply through by R.

$$(R)(240) - (R)(R) + (\cancel{R})\left(\frac{20(240)}{\cancel{R}}\right) - (R)(20) = (R)(240)$$

$$240R - R^2 + 4{,}800 - 20R = 240R$$

$$-R^2 - 20R + 4{,}800 = 0$$

Next multiply through by -1.

$$(-1)(-R^2) - (-1)(20R) + (-1)(4{,}800) = (-1)(0)$$

$$R^2 + 20R - 4{,}800 = 0$$

Factor for R.

$$(R+80)(R-60) = 0$$

$$R = -80 \text{ or } 60$$

$$R = 60$$

We choose 60 and ignore –80 because it is a negative number and time (or distance) can never be negative.

An alternative approach involves *backsolving*. The algebraic setup follows:

Slower time – faster time = 1 hour

$$\frac{240}{R} - \frac{240}{R+20} = 1 \text{ hour}$$

Now backsolve. That is, take the various answer choices and substitute them into the formula above and see which results in an answer of 1 hour. Our correct answer is 60 m.p.h.

$$\frac{240}{60} - \frac{240}{60+20} = 1$$

$$\frac{240}{60} - \frac{240}{80} = 1$$

$$4 - 3 = 1 \text{ hour}$$

6. Sixteen-Wheeler ( )

Choice B
Classification: Distance-Rate-Time Problem
Snapshot: This problem is a variation of a "two-part D-R-T" problem. Distance is a constant, although individual distances and rates and times are different. The applicable formula is: $D = (R_1 \times T_1) \times (R_2 \times T_2)$. Distance is a constant because the combined distances traveled by the two drivers will always be the same.

$$D = \text{Distance}_{\text{(Driver A)}} + \text{Distance}_{\text{(Driver B)}}$$

$$D = (\text{Rate}_1 \times \text{Time}_1) + (\text{Rate}_2 \times \text{Time}_2)$$

The distance covered by Driver A is $90(T + 1)$. The distance covered by Driver B is $80T$.

$80T + 90(T + 1) = 770$ (where T equals the time of Driver B)
$80T + 90T + 90 = 770$
$170T = 680$
$T = \frac{680}{170}$
$T = 4$ hours

Given that Driver B took 4 hours, Driver A took 5 hours (i.e., $T + 1$). We now calculate how far each person has traveled and take the difference:

Driver A:

$$D = R \times T$$

$$D = 90(T+1) = 90(4+1) = 90(5)$$

$$D = 450 \text{ kilometers}$$

Driver B:

$$D = R \times T$$
$$D = 80T = 80(4)$$
$$D = 320 \text{ kilometers}$$

Finally, 450 – 320 = 130 kilometers

Note: This problem could also have been solved by expressing "time" in terms of Driver A:

$$90T + 80(T - 1) = 770$$
$$90T + 80T - 80 = 770$$
$$170T = 850$$
$$T = \frac{850}{170}$$
$$T = 5 \text{ hours}$$

In this case, we can confirm that whereas Driver A took 5 hours, Driver B took 4 hours. Driver A drove for 5 hours at 90 kilometers per hour (450 miles). Driver B drove for 4 hours at 80 kilometers per hour (320 miles). The <u>difference</u> in distances driven is 130 kilometers.

7. Elmer ( )

Choice B
Classification: Age Problem
Snapshot: In cases where an age problem states, "Alan is twice as old as Betty," the math translation is $A = 2B$, not $2A = B$. In cases where an age problem states, "10 years from now Sam will be twice as old as Tania," the math translation is $S + 10 = 2(T + 10)$. In this latter situation, remember to add 10 years to both sides of the equation because both individuals will have aged 10 years.

First Equation:

Elmer is currently three times older than Leo.
$E = 3L$

Second Equation:

In five years from now, Leo will be exactly half as old as Elmer.
$2(L + 5) = E + 5$

Next, we substitute for the variable E in order to solve for L. That is, substitute $3L$ (First Equation) for the variable E (Second Equation).

$$2(L + 5) = 3L + 5$$
$$2L + 10 = 3L + 5$$

$-L = -5$

$(-1)(-L) = (-1)(-5)$ [multiply through by -1]

$L = 5$

Therefore, since Leo, the circus Lion, is 5 years old, this means that Elmer, the circus Elephant, is 15 years old. This calculation is derived from the first equation, $E = 3L$.

8. Three's Company ()

Choice D
Classification: Average Problem
Snapshot: If the average of eight numbers is 7, their sum must be 56. This simple revelation provides a key step in solving most average problems.

$$\text{Average} = \frac{\text{Sum of Terms}}{\text{Number of Terms}}$$

$$\text{Avg} = \frac{4(4x+3)-x}{3} = \frac{16x+12-x}{3} = \frac{15x+12}{3} = 5x+4$$

9. Fourth Time Lucky ()

Choice C
Classification: Average Problem
Snapshot: This average problem requires a solution in the form of an algebraic expression.

$$\text{Average} = \frac{\text{Sum of Terms}}{\text{Number of Terms}}$$

$$\text{Avg} = \frac{3N+(N+20)}{4}$$

$$\text{Avg} = \frac{(4N+20)}{4} = \frac{4N}{4} + \frac{20}{4} = N+5$$

Note: Since Rajeev received an average score of N points on his first 3 tests, his total points is $3N$.

10. Vacation ()

Choice D
Classification: Average Problem
Snapshot: This special type of average problem may be referred to as a "dropout problem." Specifically, we want to know how much *additional* money each individual must pay as a result of others dropping out. This problem type also requires mastery of algebraic fractions.

First Equation:

$\frac{x}{P}$ represents the amount each person was originally going to pay before the dropouts.

Second Equation:

$\frac{x}{P-D}$ represents the total amount each person has to pay after the dropouts.

Accordingly, $\frac{x}{P-D}-\frac{x}{P}$ will yield the additional amount that each person must pay. Note that the amount that each person has to pay after the dropouts is greater per person than the amount before.

Therefore we subtract the First Equation from the Second Equation, not the other way around. The algebra here requires dealing with algebraic fractions which can be a bit confusing. Multiply each term through by the common denominator of $P(P-D)$.

$$\frac{x}{P-D}-\frac{x}{P}$$

$$\frac{P(\cancel{P-D})\frac{x}{\cancel{P-D}}-\cancel{P}(P-D)\frac{x}{\cancel{P}}}{P(P-D)}$$

$$\frac{Px-\left[(P-D)x\right]}{P(P-D)}$$

$$\frac{\cancel{Px}-\cancel{xP}+Dx}{P(P-D)}$$

$$\frac{Dx}{P(P-D)}$$

Author's note: See problem 36, *Hodgepodge*, which provides another example of working with algebraic fractions.

11. Disappearing Act ( )

Choice C
Classification: Work Problem
Snapshot: This problem is a "walk-away work problem." Two people (or two machines) will set to work on something and, after a stated period of time, one of the individuals gets up and leaves (or one of the machines breaks down), forcing the remaining person (or machine) to finish the task.

Since an hour is an easy unit to work with, think in terms of how much of the task each person working alone could complete in just one hour. Deborah could do the job in 5 hours, so she does $\frac{1}{5}$ of it in an hour; Tom could do the job in 6 hours, so he does $\frac{1}{6}$ of it in one hour. Working together for 2 hours, they complete $\frac{11}{15}$ of that job, which leaves $\frac{4}{15}$ of the task for Deborah to complete alone. Deborah can complete $\frac{4}{15}$ of the task in $1\frac{1}{3}$ hours.

The solution unfolds in three steps:

i) Amount of work they both will do in 2 hours:

$$2\left(\frac{1}{5}+\frac{1}{6}\right)=2\left(\frac{6}{30}+\frac{5}{30}\right)=2\left(\frac{11}{30}\right)=\frac{22}{30}=\frac{11}{15}$$

ii) Amount of work left to do:

$$1-\frac{11}{15}=\frac{4}{15}$$

iii) Time it takes Deborah to complete the task alone:

$$\text{Time}=\frac{\text{Amount of Work}}{\text{Deborah's Work Rate}}$$

$$T=\frac{\frac{4}{15}}{\frac{1}{5}}=\frac{4}{15}\times\frac{5}{1}=\frac{20}{15}=\frac{4}{3}=1\tfrac{1}{3}\text{ hours}$$

Author's note: The general formula for work problems is "$\frac{1}{A}+\frac{1}{B}=\frac{1}{T}$," where A and B represent the time it takes a given person or machine to individually complete a task and T represents the total time it takes both persons or machines to complete the task working together but independently.

12. Exhibition ( )

Choice C
Classification: Work Problem
Snapshot: For problems which involve the work rates for a group of individuals (or machines), calculate first the work rate for a single person (or machine) and then multiply this rate by the number of persons (or machines) in the new group. The time necessary to complete the new task will be 1 divided by this number.

Solution in four steps:

i) Find how much of the job 70 workers could do in 1 hour.

Result: If 70 workers can do the job in 3 hours then the 70 workers can do $\frac{1}{3}$ of the job in one hour.

ii) Find out how much of the job a single worker can do in 1 hour.

Result: $\frac{1}{70} \times \frac{1}{3} = \frac{1}{210}$. This is the hourly work rate for an individual worker.

iii) Multiply this rate by the number of new workers.

Result: $30 \times \frac{1}{210} = \frac{30}{210} = \frac{1}{7}$. This is the work rate for the group of new workers.

iv) Take the reciprocal of this number and voila—the answer!

Result: $\frac{1}{7}$ becomes $\frac{7}{1} = 7$ hours

Alternative Solution:

If 70 men could do the work in 3 hours, then how long would it take 30 men to the do the same job?

$$30H = 70 \times 3$$
$$30H = 210$$
$$H = \frac{210}{30} = 7 \text{ hours}$$

13. Legal ( )

Choice B
Classification: Work Problem
Snapshot: Again, the key is to think in terms of a *work rate* for a single individual. Next, multiply this figure by the total number of new group members to find a "group rate," and finally, divide this number by 1 to find total hours.

Five steps:

i) Find how much of the job 4 junior lawyers could do in 1 hour.

If 4 junior lawyers can do the job in 5 hours then the 4 junior lawyers can do $\frac{1}{5}$ of the job in one hour.

ii) Find out how much of the job a single worker can do in 1 hour.

Result: $\frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$. This is the work rate for a single junior lawyer.

iii) Adjust this work rate for the rate of legal assistants versus junior lawyers.

Result: $\frac{1}{20} \times \frac{2}{3} = \frac{2}{60}$. This is the adjusted work rate for a single legal assistant.

iv) Multiply this rate by the number of legal assistants.

Result: $3 \times \frac{2}{60} = \frac{6}{60}$. This is the work rate for the group of three legal assistants.

v) Take the reciprocal of this number and voila—the answer!

Result: $\frac{6}{60}$ becomes $\frac{60}{6} = 10$ hours

Alternative Solution:

$$3 \times \frac{2}{3} \times H = 4 \times 5$$

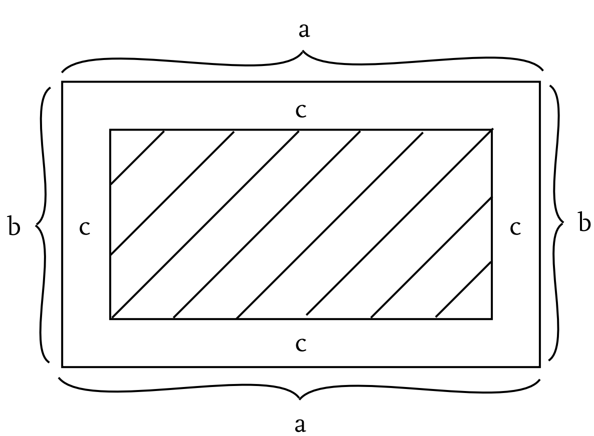
$$\frac{6}{3} H = 20$$

$$\frac{\cancel{3}}{\cancel{6}} \times \frac{\cancel{6}}{\cancel{3}} H = \frac{3}{6} \times 20$$

$$H = \frac{60}{6} = 10 \text{ hours}$$

14. Persian Rug ( )

Choice E
Classification: Picture Frame or Border Problem
Snapshot: Don't forget that a frame or border surrounding a picture or carpet contains a border on all sides.



The key is to subtract the area of the entire rug (rug plus border) from the area of the rug itself (rug minus border).

i) Find the area of the entire rug (including the border) in square inches.
Answer: $a \times b$ or ab.

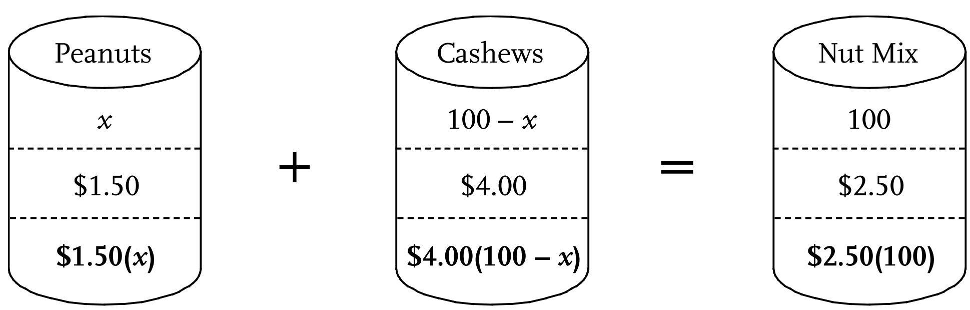
ii) Find the area of the rug itself (without the border) in square inches.
Answer: $(a - 2c)(b - 2c)$.

iii) Find the area of the strip (in square inches) by subtracting the area of the rug (excluding border) from the area of entire rug (including border).
Answer: $ab - (a - 2c)(b - 2c)$.

The answer is not choice D which assumes that the frame is only on one side of the picture. A border on a rectangle or square object occurs on all sides of the object. Answer choice C represents the difference in perimeters. This would have been the correct answer had the question asked, "Which algebraic expression below represents the positive difference in the measure of the perimeter of the rug and the rug design?"

15. Nuts ( )

Choice A
Classification: Mixture Problem
Snapshot: This is a dry mixture. We need to calculate the amounts of two different nut mixtures to arrive at a final mixture.



$$\$1.50(x) + \$4.00(100 - x) = \$2.50(100)$$
$$\$1.50x + \$400 - \$4.00x = \$250$$
$$-\$2.5x = -\$150$$
$$(-1)(-\$2.5x) = (-1)(-\$150)$$
$$\$2.5x = \$150$$
$$x = \frac{\$150}{\$2.5}$$
$$x = 60 \text{ pounds of peanuts}$$

Therefore, $100 - 60 = 40$ pounds of cashews

The following is an alternative solution using a *two-variables, two-equations* approach. Substitute for one of the variables, x or y, and solve. Here the variable x represents peanuts while the variable y represents cashews.

i) $\$1.50x + \$4.00y = \$2.50(100)$
ii) $x + y = 100$

"Put equation ii) into equation i) and solve for y." That is, substitute for the variable *"x"* in equation i), using $x = 100 - y$ per equation ii). Although we could substitute for either variable, we prefer to substitute for x and solve for y given that the final answer is expressed in terms of cashew nuts.

$\$1.50(100 - y) + \$4.00y = \$2.50(100)$
$\$150 - \$1.5y + \$4.00y = \$250$
$-\$100 = -\$2.5y$
$\$2.5y = \$100$
$y = 40$ pounds of cashews

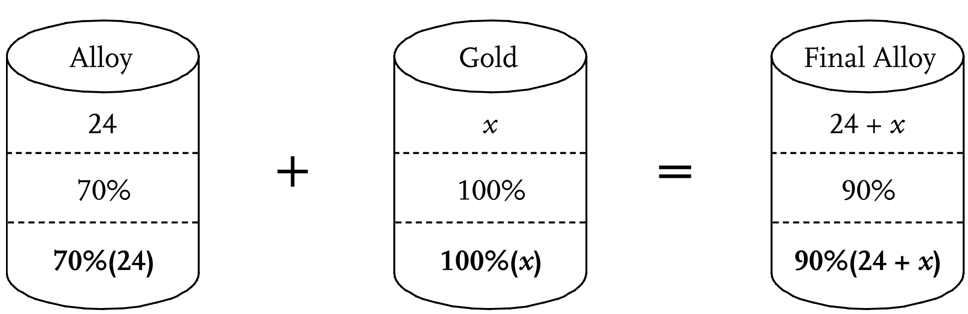
Author's note: Mixture problems are best solved using the *barrel method* which summarizes information similar to a 3-row by 3-column table.

This problem garners one-chili rating partly because it is easy to backsolve, especially given the fact that the answer is choice A and we would likely begin choosing answer choices beginning with choice A. From the information given in the problem, we know we are looking for a final mixture that costs \$2.50 per pound. We also know the individual prices per pound. Choice A tells us that we have 40 pounds of cashews and, by implication, 60 pounds of peanuts. Will this give us an answer of \$2.50? Yes it does, and the answer choice A is confirmed.

40 pounds	×	\$4.00	=	\$160.00		Cashews
60 pounds	×	\$1.50	=	\$90.00		Peanuts
				\$250.00		
Total pounds				100		
Price per pound				\$2.50		

16. Gold ( )

Choice E
Classification: Mixture Problem
Snapshot: This is a dry mixture, which involves percentages. We need to calculate the amount of pure gold that needs to be added to arrive at a final alloy. When adding pure gold, we use 100%. If we were to add a pure non-gold alloy, the percentage would be 0%.



$70\%(24) + 100\%(x) = 90\%(24 + x)$

$16.80 + x = 21.6 + 0.90x$

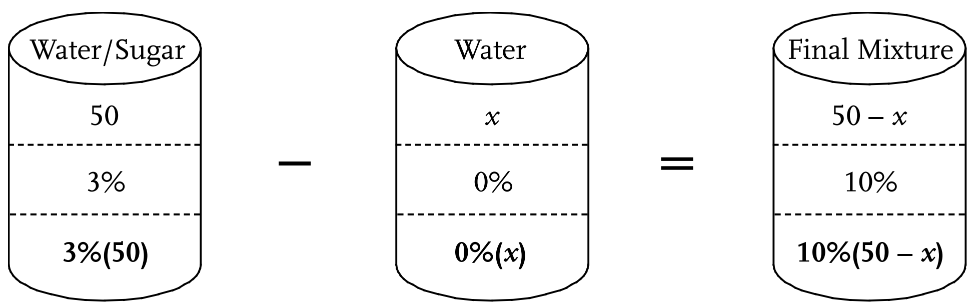
$0.1x = 4.8$

$$x = \frac{4.8}{0.1}$$
$$x = 48 \text{ ounces of pure gold}$$

17. Evaporation ( )

Choice A
Classification: Mixture Problem
Snapshot: This is a wet mixture. We need to calculate the amount of pure water that needs to be subtracted to arrive at a final solution. The percentage for pure water is 0 percent because pure water (whether added or subtracted) lacks any "mixture." This causes the middle term in the equation to drop out.

This is a wet mixture. We need to calculate the amount of pure water that needs to be subtracted to arrive at a final solution.



$$3\%(50) - 0\%(x) = 10\%(50 - x)$$
$$1.5 - 0 = 5 - 0.10x$$
$$-3.5 = -0.10x$$
$$0.10x = 3.5$$
$$x = \frac{3.5}{0.1}$$
$$x = 35 \text{ liters of pure water}$$

Author's note: In the above equation, we simply reverse the equation and change the signs, so that "$-3.5 = -0.10x$" becomes "$0.10x = 3.5$." Another way to view this practice algebraically is as follows:

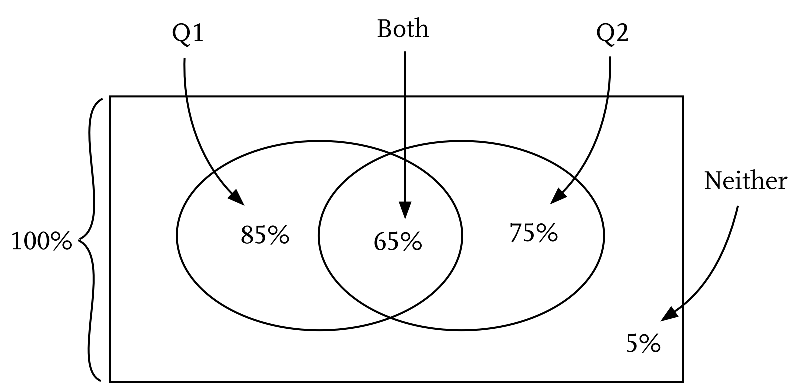
$-3.5 = -0.10x$
$(-1)(-3.5) = (-1)(-0.10x)$ [multiply both sides by –1]
$3.5 = 0.10x$
$0.10x = 3.5$ [simply switch terms around; that is, if A = B, then B = A]

In short, when solving for a single variable such as x, our practical goal is to get x on one side of the equation and all other terms on the opposite side of the equation. Typically, this involves isolating x on the left-hand side of the equation while placing all the other terms on the right-hand side of the equation. There are two useful concepts to keep in mind when manipulating elements of a formula to in order to achieve this objective. The first is that bringing any number (or term) across the equals sign, changes the sign of that number (or term). That is, positive numbers become negative and negative numbers become positive. The second useful concept is that whatever we do to one term in an equation, we must do to every other term in that same equation (in order to maintain the equivalent value of all terms in the equation). So in the above equation we may choose to multiply each side of the equation by –1 in order to cancel the negative signs. We do this because we always want to solve for a positive x value.

18. Standardized Test ( )

Choice B
Classification: Group Problem
Snapshot: This category of problem deals most often with situations which are not mutually exclusive and will therefore contain overlap; this overlap must not be double counted, otherwise the number of members or items in a group will exceed 100 percent due to mutual inclusivity. In short, group problems will either give you *neither* and ask for *both* or give you *both* and ask for *neither.*

The Venn Diagram below lends a pictorial. Note that 100 percent is what is in the "box"; it includes Q1 & Q2 and *neither,* but it must not include *both* because *both* represents overlap that must be subtracted out; otherwise the overlap will be double counted. In other words, either Q1 may include the 65% as part of its 85% or Q2 may include the 65% as part of its 75%, but Q1 and Q2 cannot both claim it. Another way to view this problem is to break up analytically as follows: The number of students who only got Q1 correct is 20% (85% – 65%) while the number of students who only got Q2 correct is 10% (75% – 65%). Since 65% of students got both questions correct, the number of students who got one or the other question correct is: 20% + 10% + 65% = 95%; 5% of students got neither question correct.



The following is the mathematical solution to this problem.

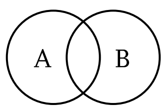
Two-Groups Formula	*Solve*
+ Group A	+ 85%
+ Group B	+ 75%
– Both	– x%
+ Neither	+ 5%
Total	100%

Calculation:

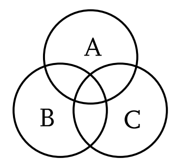
Group A + Group B – Both + Neither = Total
$85\% + 75\% - x + 5\% = 100\%$
$165\% - x = 100\%$
$x = 65\%$

Author's note: Some group problems involve two overlapping circles and some involve three overlapping circles. The formulas below are the key to solving these types of problems with a minimum of effort.

Summary of Templates:



Two-Groups Formula		Solve
Add:	Group A	x
Add:	Group B	x
Less:	Both	$<x>$
Add:	Neither	x
	Total	xx



Three-Groups Formula		Solve
Add:	Group A	x
Add:	Group B	x
Add:	Group C	x
Less:	A & B	$<x>$
Less:	A & C	$<x>$
Less:	B & C	$<x>$
Add:	All of A & B & C	x
Add:	None of A or B or C	x
	Total	xx

19. Language Classes ( )

Choice D
Classification: Group Problem
Snapshot: This type of Group Problem requires that we express our answer in algebraic form.

This problem is more difficult and garners a three-chili rating in so far as it requires an answer that is expressed in terms of an algebraic expression.

Use the classic Two-Groups Formula:

Group A + Group B – Both + Neither = Total

Applied to the problem at hand:

Spanish + French – Both + Neither = Total Students
$S + F - B + N = X$
$N = X + B - F - S$

Therefore, expressed as a percent:

$$\text{Neither} = 100\% \times \frac{X + B - F - S}{X}$$

20. German Cars ( )

Choice C
Classification: Group Problem
Snapshot: This problem involves three overlapping circles. There are two ways to solve this problem. One employs the "three-groups" formula, while the other involves using the Venn-diagram approach. For the purpose of answering this question in two minutes, it is highly recommended that candidates use the "three-groups" formula.

I. Three-Groups Formula

The three-groups formula is preferable because it is clearly fastest, relying directly on the numbers found right in the problem. In short, the total number of individuals owning a single car are added together, the number of individuals owning exactly two of these cars is subtracted from this number, and, finally, the number of people owning all three of these cars are added back. The calculation follows:

Three-Groups Formula		**Solve**
Add:	BMW	45
Add:	Mercedes	38
Add:	Porche	27
Less:	BMW & Mercedes	<15>
Less:	BMW & Porche	<12>
Less:	Mercedes & Porche	<8>
Add:	BMW & Mercedes & Porche	5
Add:	None of BMW & Mercedes & Porche	0
	Total	80

Calculation:

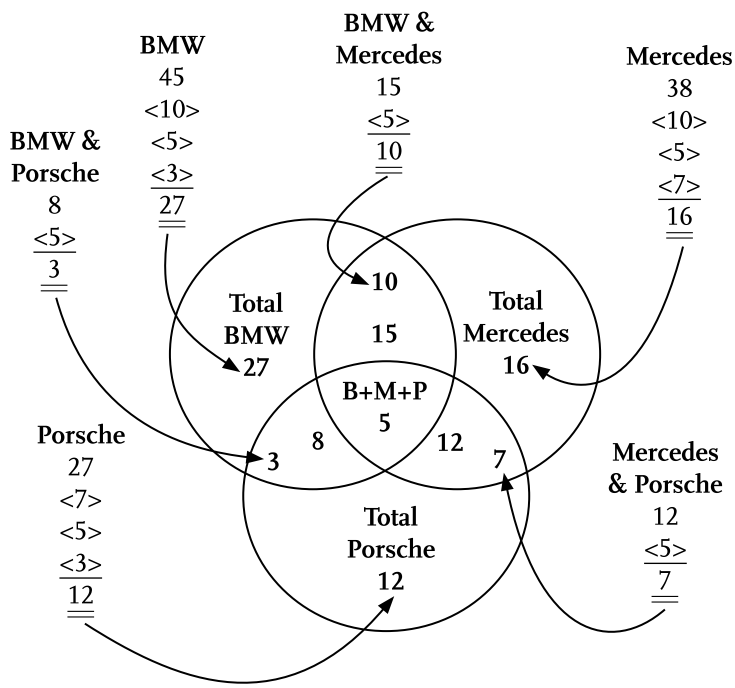
B + M + P – BM – BP – MP + BMP + None = Total
45 + 38 + 27 – 15 – 12 – 8 + 5 + 0 = x
x = 80

II. The Venn-diagram Approach

The Venn-diagram approach is highly analytical and requires breaking the problem down into non-overlapping areas while finding individual values for all seven areas. That's right!—seven distinct areas are created when three groups overlap.

27 + 16 + 12 + 10 + 7 + 3 + 5 = 80

BMW only + Mercedes only + Porsche only – [(BMW & Mercedes) – (Mercedes & Porsche) – (BMW & Porsche)] + (BMW & Mercedes & Porsche).



21. Single ( )

Choice D
Classification: Matrix Problem
Snapshot: A matrix can be used to summarize data, particularly data that is being contrasted across two variables and which can be sorted into four distinct outcomes. Use a table with nine boxes and fill in the data.

Two-thirds of the women are single (i.e., $\frac{20}{30} = \frac{2}{3}$). For this problem, assume for simplicity's sake that there are 100 students in the course and fill in the given information, turning percentages into numbers. If 70% of the students are male then 30% must be female. If we assume there are 100 students then 70 are male and 30 are female. Note that if two-sevenths of the male students are married, then 20 male students are married; that is, two-sevenths of 70 students equals 20 students.

First, fill in the information derived directly from the problem:

	Male	Female	
Married	20		30
Single		?	
	70	?	100

Second, complete the matrix by filling in the remaining information, ensuring that numerical data totals both down and across. The shaded boxes are those which contain the specific data to solve this problem.

	Male	Female	
Married	20	10	30
Single	50	20	70
	70	30	100

22. Batteries ( )

Choice A
Classification: Matrix Problem
Snapshot: To solve difficult matrix problems try "picking numbers," particularly the number 100, if possible.

To obtain the percent of batteries sold by the factory that are defective, we fill in the information per the matrix below to obtain $\frac{3}{75}$ or $\frac{1}{25}$ or 4%. As in the previous problem, the technique of picking of the number "100" greatly simplifies the task at hand.

First, fill in the information directly deducible from the problem:

	Defective	Not Defective	
Rejected		$\frac{1}{10}(80) = 8$	$\frac{1}{4}(100) = 25$
Not Rejected	?		?
	$\frac{1}{5}(100) = 20$	80	100

Second, complete the matrix. Information appearing in shaded boxes is the key to solving the problem.

	Defective	Not Defective	
Rejected	17	8	25
Not Rejected	3	72	75
	20	80	100

23. Experiment ( )

Choice A
Classification: Matrix Problem
Snapshot: This is a difficult, odd-ball matrix problem. Although it is possible to employ a traditional matrix, a guesstimate must be made regarding one of the numbers.

There are two ways to solve this problem: the "picking numbers" approach and the "algebraic" approach.

I. Picking Numbers Approach

Let's pick numbers. Say the total number of rats that are originally alive is 100, of which 60 are female and 40 are male. Let's say that 50 rats die, which means 15 were female and 35 were male. Calculation for dead rats is: 30% × 50 = 15 female rats versus 70% × 50 = 35 male rats.

	Male	Female	
Rats that Died	35	15	*50
Rats that Lived			
	40	60	100

Note that "*50" is a mere guesstimate.

Thus, the ratio of death rate among male rats to the death rate among female rats is calculated as follows:

Complete the matrix. The shaded boxes represent information that is key to solving the problem.

	Male	Female	
Rats that Died	35	15	*50
Rats that Lived	5	45	**50
	40	60	100

Note that "**50" is also an estimate (and a plug number). Data about the number of rats that lived is not useful in the problem at hand; the focus is on the number of rats that died.

Ratio of the number of male rats that died to the number of female rats that died:

$$\frac{\dfrac{\text{male rat deaths}}{\text{male rats (total)}}}{\dfrac{\text{female rat deaths}}{\text{female rats (total)}}} = \frac{\dfrac{35}{40}}{\dfrac{15}{60}} = \frac{35}{40} \times \frac{60}{15} = 7:2$$

II. Algebraic Approach

$$\frac{\dfrac{0.7 \text{ died (male)}}{0.4 \text{ total (male)}}}{\dfrac{0.3 \text{ died (female)}}{0.6 \text{ total (female)}}} = \frac{0.7\ \cancel{\text{died}}}{0.4\ \cancel{\text{total}}} \times \frac{0.6\ \cancel{\text{total}}}{0.3\ \cancel{\text{died}}} = \frac{\cancel{0.42}^{\,7}}{\cancel{0.12}^{\,2}} = \frac{7}{2} = 7:2$$

The trap answer, per choice B, may be calculated in two ways. The first involves using numbers we previously obtained through the picking numbers approach. In this case: 35/15 = 7:3. However, this ratio is not correct because it involves dividing the estimated number of dead male rats by the number of dead female rats. We need to instead divide the death rate of male rats by the death rate of female rats.

Another way of obtaining trap answer choice B is as follows:

$$\frac{\dfrac{\text{male rat deaths}}{\text{male rats (total)}}}{\dfrac{\text{female rat deaths}}{\text{female rats (total)}}} = \frac{\dfrac{70\% \times 40\%}{40\%}}{\dfrac{30\% \times 60\%}{60\%}} = \frac{\dfrac{28\%}{40\%}}{\dfrac{18\%}{60\%}} = \frac{28\%}{40\%} \times \frac{60\%}{18\%} = 7:3$$

The problem here involves multiplying the respective death rates for male and female rats by the percentage of male and female rats. However, this is erroneous since we do not know how many rats died. In other words, we are dealing with two different groups of rats. The first group represents the total number of rats and the second group represents the rats that died. No direct link can be drawn between these two groups so we cannot directly multiply these percentages.

24. Garments ( )

Choice B
Classification: Price-Cost-Volume-Profit Problem
Snapshot: When Price-Cost-Volume-Profit problems are presented in the form of algebraic expressions, it is best to first find a per unit figure—dollar per unit or dollar per individual, usage per unit or usage per individual.

This problem requires us to calculate "number of units or volume." Start with the basic "cost" formula and solve for "number of units" as follows:

$$\text{Total Cost} = \text{Cost}_{\text{ per unit}} \times \text{Number of units}$$

$$\text{Therefore, Number units} = \frac{\text{Total Cost}}{\text{Cost}_{\text{ per unit}}}$$

$$\text{Number of units} = \frac{t \text{ dollars}}{\dfrac{d \text{ dollars}}{s \text{ shirts}}}$$

$$\text{Number of units} = t\ \cancel{\text{dollars}} \times \frac{s \text{ shirts}}{d\ \cancel{\text{dollars}}}$$

$$\text{Number of units} = \frac{ts}{d} \text{ shirts}$$

In summary, since the "price per unit" equals d/s, we divide t by d/s to arrive at ts/d.

25. Pet's Pet Shop ( )

Choice B
Classification: Price-Cost-Volume-Profit Problem
Snapshot: The key is to find a "usage per unit" figure, then work outward.

This problem requires that we calculate a "usage per unit" figure.

$$\text{Cups per bird per day} = \frac{35 \text{ cups}}{\dfrac{15 \text{ birds}}{7 \text{ days}}}$$

$$\text{Cups per bird per day} = \frac{35 \text{ cups}}{15 \text{ birds}} \times \frac{1}{7 \text{ days}}$$

$$\text{Cups per bird per day} = \frac{1}{3} \text{ cups per bird per day}$$

Therefore the number of cups of bird seed to feed 9 birds for 12 days is:

$$\frac{1}{3} \text{ cups per bird per day} \times 9 \text{ birds} \times 12 \text{ days} = 36 \text{ cups}$$

26. Sabrina ( )

Choice C
Classification: Price-Cost-Volume-Profit Problem
Snapshot: This problem tests break-even point in terms of total revenue.

The difference between Sabrina's current base salary, \$85,000, and \$45,000 is \$40,000. Divide \$40,000 by 15%(\$1,500) to get 177.77 sales. In the equation below, x stands for the number of sales.

$$\text{Revenue}_{\text{Option 1}} = \text{Revenue}_{\text{Option 2}}$$

$$\$85{,}000 = \$45{,}000 + 0.15(\$1{,}500)(x)$$
$$\$85{,}000 - \$45{,}000 = 0.15(\$1{,}500)(x)$$
$$\$40{,}000 = 0.15(\$1{,}500)(x)$$
$$\$40{,}000 = \$225x$$
$$\$225x = \$40{,}000$$
$$x = \frac{\$40{,}000}{\$225}$$
$$x = 177.77$$

Therefore, $x = 178$ unit sales

Author's note: When dividing \$40,000 by \$225 (commission) per sale, the dollar signs cancel, leaving the answer as the number of sales (or units).

Don't be tricked by tempting wrong answer choice B. A total of 177 sales isn't enough to break even. This number must be rounded up to 178 in order to avoid losing money. The actual number of sales is discrete, and can only be represented by whole numbers, not decimals.

27. Delicatessen ( )

Choice B
Classification: Price-Cost-Volume-Profit Problem
Snapshot: This type of problem shows how to calculate gross profit (or gross margin) when expressed in algebraic terms.

A variation of the "profit" formula is:

$$\text{Profit} = (\text{Price}_{\text{per unit}} \times \text{No. of units}) - (\text{Cost}_{\text{per unit}} \times \text{No. of units})$$

$$\text{Profit} = s\,\frac{\text{dollars}}{\text{pound}} \times \left(p_{\text{ pounds}} - d_{\text{ pounds}}\right) - \left(c\,\frac{\text{dollars}}{\text{pound}} \times p_{\text{ pounds}}\right)$$

$$\text{Profit} = s(p-d) - cp$$

Author's note: First, in terms of units, "pounds" cancel out and leave dollars, which of course is the unit measurement for profit. Second, "gross profit" is *sales revenue* minus *product cost*. Sales revenue is price per unit multiplied by the number of units: $s(p-d)$. Product cost is cp. Therefore, gross profit is $s(p-d) - cp$. The algebraic expressions cp and pc are of course identical.

28. Prototype ( )

Choice B
Classification: Price-Cost-Volume-Profit Problem
Snapshot: This problem highlights the concepts of *efficiency* and *cost efficiency*.

First, we set up the problem conceptually:

$$\left(1.0\,\frac{\text{dollar}}{\cancel{\text{gallon}}} \times 1.0\,\cancel{\text{gallon}}\right) - x \text{ dollar savings} = \left(1.2\,\frac{\text{dollar}}{\cancel{\text{gallon}}} \times \frac{5}{9}\,\cancel{\text{gallons}}\right)$$

Question: Where does the fraction $\frac{5}{9}$ come from? An 80 percent increase in efficiency can be expressed as $\frac{180\%}{100\%}$ or $\frac{9}{5}$. The reciprocal of $\frac{9}{5}$ is $\frac{5}{9}$. This means that the P-Car needs only five-ninths as much fuel to drive the same distance as does the T-Car.

Second, we solve for x, which represents the cost savings when using the P-Car.

Let's convert \$1.2 to $\frac{6}{5}$ for simplicity.

$$\$(1.0)(1.0) - \$x = \$\left(\frac{6}{5}\right)\left(\frac{5}{9}\right)$$

$$\$1.0 - \$x = \$\left(\frac{6}{\cancel{5}}\right)\left(\frac{\cancel{5}}{9}\right)$$

$$\$1 - \$x = \$\frac{2}{3}$$

$$-\$x = -\$1 + \$\frac{2}{3}$$

$$-\$x = -\$\frac{1}{3}$$

$$(-1)(-\$x) = (-1)(-\$\tfrac{1}{3}) \qquad \text{[multiply both sides by } -1\text{]}$$

$$\$x = \$\frac{1}{3} \qquad \text{[dollar signs cancel]}$$

$$x = \frac{1}{3} = 33\tfrac{1}{3}\%$$

Since we save one-third of a dollar for every dollar spent, our percentage savings is $33\frac{1}{3}\%$. Therefore, although the cost of gas for the T-Car is more expensive, it results in an overall cost efficiency.

For the record, whereas 80% represents how much more <u>efficient</u> the P-Car is compared to the T-Car, the correct answer, $33\frac{1}{3}$ % represents how much more <u>cost efficient</u> the P-Car is compared to the T-Car.

29. Lights ( )

Choice D
Classification: Least-Common-Multiple Word Problem
Snapshot: This problem highlights the use of prime factorization in solving L-C-M word problems. To find the point at which a series of objects "line up," find the Least Common Multiple of the numbers involved.

There are two ways to solve L-C-M problems including prime factorization and trial and error. First the *Prime Factor Approach*. Find least common multiple of 1 minute, 1 minute, 3 minutes, 1 minute, and 4 minutes. You should get 12 minutes. Therefore, every twelve minutes all lights will flash together. The columns below are useful in organizing data; the final column contains key information.

I. Prime Factor Method

Color	Prime Factors
Red	$2 \times 2 \times 5 = 20$ seconds
Blue	$2 \times 3 \times 5 = 30$ seconds
Green	$3 \times 3 \times 5 = 45$ seconds
Orange	$2 \times 2 \times 3 \times 5 = 60$ seconds
Yellow	$2 \times 2 \times 2 \times 2 \times 5 = 80$ seconds

Therefore: $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 720$ seconds

II. Trial-and-Error Method

Color	Flash Time	Cycles	Time
Red	20 seconds	3 cycles	1 minute
Blue	30 seconds	2 cycles	1 minute
Green	45 seconds	4 cycles	3 minutes
Orange	60 seconds	1 cycle	1 minute
Yellow	80 seconds	3 cycles	4 minutes

The trial-and-error method pivots on cycles. "How can the number of seconds for "Flash Time" be turned into cycles?" Observing the numbers in the "Time" column, you ask, "What is the least common multiple of 1 minute, 1 minute, 3 minutes 1 minute, and minutes?" "Voila!" The answer is 12 minutes.

30. Hardware ( )

Choice A
Classification: General Algebraic Word Problem
Snapshot: This problem further highlights the need to translate words into math. If we have twice as many pencils as crayons, the algebraic expression is $P = 2C$, not $2P = C$.

The total weight of 2 hammers and 3 wrenches is one-third that of 8 hammers and 5 wrenches:

$$3(2H + 3W) = 8H + 5W$$
$$6H + 9W = 8H + 5W$$
$$4W = 2H$$
$$W = \frac{2}{4}H$$
$$W = \frac{1}{2}H$$

Choosing the correct answer is perhaps the trickiest step because the mathematical result may seem counterintuitive and may be interpreted in reverse. Since $W = \frac{1}{2}H$ or $2W = H$, this means that two wrenches are as heavy as one hammer. Stated another way, a single wrench is half the weight of one hammer.

31. Snooker ( )

Choice C
Classification: General Algebraic Word Problem
Snapshot: The classic way to solve GMAT algebraic word problems is to identify two distinct equations and then substitute for one of the variables.

Set-up: x = general admission seat tickets and y = VIP seat tickets

First equation:

$x + y = 320$
$y = 320 - x$ Solve for y in terms of x.

Second equation:

$\$15x + \$45y = \$7,500$
$\$x + \$3y = \$500$ Simplify by dividing each term by the number 15.

"Put the first equation into the second equation." That is, we substitute for the variable y in the first equation and solve for x.

$$\$x + \$3(320 - x) = \$500$$
$$\$x + \$960 - \$3x = \$500$$
$$-\$2x = -\$460$$
$$(-1)(-\$2x) = (-1)(-\$460)$$
$$\$2x = \$460$$
$$x = \frac{\$460}{\$2}$$
$$x = 230 \text{ tickets}$$

Therefore, using the first equation (or, equally, the second equation), we substitute, $x = 230$, and solve for VIP seats:

$$230 + y = 320$$
$$y = 90$$

Finally, the difference between 230 and 90 is 140. This represents how many more general admission seat tickets were sold than VIP seat tickets.

32. Chili Paste ( )

Choice D
Classification: General Algebraic Word Problem
Snapshot: This problem contrasts one-variable versus two-variable problem-solving approaches. Four scenarios are possible in terms of this particular problem: x = small cans and y = large cans

Scenario 1: $\frac{x}{15} - \frac{x}{25} = 40$ (where x = weekly needs)

$x = 1{,}500$ ounces
Thus, $1{,}500 \div 15 = 100$ small cans

This problem is solved using a single variable; the weekly needs of the restaurant is expressed in terms of x.

Scenario 2: $15x = 25(x - 40)$ (where x = small cans)
$x = 100$ small cans

Scenario 3: $25y = 15(y + 40)$ (where y = large cans)
$y = 60$ large cans
$x = 100$ small cans

Scenario 4: i) $x = y + 40$ (where x = small cans)

ii) $y = \frac{15}{25}x$ (or $x = \frac{25}{15}y$)

Substituting equation ii) into equation i) above:

Thus, $x = \frac{3}{5}x + 40$
$x = 100$ small cans

The above is a classic two-variable, two equations approach. This last scenario is tricky because the second equation utilizes a relationship involving volume, i.e., $\frac{15}{25}$ ounces.

Author's note: *Chili Paste* is an interesting problem and one which highlights multiple solutions. That's what's really interesting about math—"you can get downtown by taking many different roads." In other words, there are often multiple approaches to use to arrive at one single correct answer.

33. Premium ( )

Choice E
Classification: General Algebraic Word Problem
Snapshot: This problem involves three variables and requires an answer expressed in terms of a third variable, in this case y.

Likely the easiest way to solve this problem is to identify two equations and substitute for one of the variables.

First Equation:

$5P = 6R$
The price of 5 kilograms of premium fertilizer is the same as the price of 6 kilograms of regular fertilizer.

Second Equation:

$P - y = R$ or $R = P - y$
The price of premium fertilizer is y cents per kilogram more than the price of regular fertilizer.

Now we substitute for R in the first equation and solve for P:

$$5P = 6(P - y)$$
$$5P = 6P - 6y$$
$$-P = -6y$$
$$(-1)(-P) = (-1)(-6y)$$
$$P = 6y$$

34. Function ()

Choice B
Classification: Function Problem
Snapshot: This problem focuses on how to work with compound functions. A function is a process that turns one number into another number. Usually this involves just plugging one number into a formula. Although it is not the only variable that can be used, the letter "f" is commonly used to designate a function.

Let's refer to the two equations as follows:

First equation: $f(x) = \sqrt{x}$

Second equation: $g(x) = \sqrt{x^2 + 7}$

Start by substituting "3" into the second equation:

$f(g(x))$ means apply g first, and then apply f to the result.

$$g(x) = \sqrt{x^2 + 7}$$
$$g(3) = \sqrt{(3)^2 + 7} = \sqrt{9 + 7} = \sqrt{16} = 4$$

Next, substitute "4" into the first equation:

$$f(x) = \sqrt{x}$$
$$f(4) = \sqrt{4} = 2$$

Author's note: Here are two additional function problems.

Q1: Assuming identical information with respect to the original problem, what would be the solution to $g(f(9))$?

$g(f(x))$ means apply f first, and then apply g to the result.

First, simply substitute "9" into the first equation:

$$f(x) = \sqrt{x}$$
$$f(9) = \sqrt{9} = 3$$

Second, substitute "3" into the second equation:

$$g(x) = \sqrt{x^2 + 7}$$

$$g(3) = \sqrt{x^2+7} = \sqrt{(3)^2+7} = \sqrt{9+7} = \sqrt{16} = 4$$

Q2: What is $f(x)g(x)$ if $f(9) = \sqrt{x}$ and $g(3) = \sqrt{x^2+7}$?

$f(x)g(x)$ means apply f and g separately, and then multiply the results.

$$f(9) = \sqrt{x} = \sqrt{9} = 3$$
$$g(3) = \sqrt{x^2+7} = \sqrt{(3)^2+7} = \sqrt{16} = 4$$

Result: $f(x)g(x) = 3 \times 4 = 12$

35. Rescue ( )

Choice D
Classification: Algebraic Fraction Problem
Snapshot: This problem highlights the "factoring out of a common term" as a key to solving algebraic fraction problems.

$$a(c-d) = b-d$$
$$ac - ad = b - d$$
$$d - ad = b - ac$$
$$d(1-a) = b - ac$$
$$d = \frac{b-ac}{1-a}$$

36. Hodgepodge ( )

Choice C
Classification: Algebraic Fraction Problem
Snapshot: This problem tests the ability to deal with common denominators in solving algebraic fraction problems.

irst, we need to simplify the expression $1 - \frac{1}{h}$, which can be recast as $\frac{1}{1} - \frac{1}{h}$. The key is to pick a common denominator for each individual fraction and the product of both denominators can serve as the common denominator. In this case, "1h" will serve as the common denominator for "h" and "1":

$$1 - \frac{1}{h} = \frac{1}{1} - \frac{1}{h} = \frac{(\cancel{1}h)\frac{1}{\cancel{1}} - \frac{1}{\cancel{h}}(1\cancel{h})}{(1)(h)} = \frac{h-1}{h}$$

Now the simplified calculation becomes:

$$\frac{\frac{1}{h}}{\frac{h-1}{h}} = \frac{1}{\cancel{h}} \times \frac{\cancel{h}}{h-1} = \frac{1}{h-1}$$

Likely, the trickiest step with this problem is simplifying the fraction in the denominator of the fraction. Take this very simple example:

$$\frac{1}{2} - \frac{1}{3}$$

$$\frac{{}^{3}\cancel{6}\left(\frac{1}{\cancel{2}}\right) - {}^{2}\cancel{6}\left(\frac{1}{\cancel{3}}\right)}{(2)(3)}$$

$$\frac{3-2}{6} = \frac{1}{6}$$

In this simple example, we instinctively place the difference (that is, the integer 1) over the common denominator. It is easy to forget this step when dealing with the more difficult algebraic expression presented in this problem.

37. Mirage ( )

Choice D
Classification: Fractions and Decimals
Snapshot: This problem tests the ability to determine fraction size in a conceptual way, without the need to perform calculations.

Whenever we add the same number to both the numerator and denominator of a fraction less than 1, we will always create a bigger fraction. In this problem we are effectively adding 1 to both the numerator and denominator. Take the fraction $\frac{1}{2}$ for example. If we add 1 to both the numerator and denominator of this fraction, the fraction becomes conspicuously larger.

$$\frac{1}{2} = 50\% \qquad \frac{1+1}{2+1} = \frac{2}{3} = 66\tfrac{2}{3}\%$$

Therefore, 50% becomes $66\frac{2}{3}\%$.

This memorable example adds 1 million to both the numerator and denominator of a fraction less than 1:

$$\frac{1}{2} = 50\% \qquad \frac{1+1{,}000{,}000}{2+1{,}000{,}000} = \frac{1{,}000{,}001}{1{,}000{,}002} \cong 100\%$$

Therefore, 50% becomes almost 100%.

Alternatively, adding the same number to both the numerator and denominator of a fraction greater than 1 will always result in a smaller fraction.

Which of the following has the greatest value?

A) $\frac{11}{10}$ B) $\frac{5}{4}$ C) $\frac{8}{7}$ D) $\frac{22}{21}$ E) $\frac{6}{5}$

Choice B has the greatest value while choice D has the smallest value.

38. Deceptive ( )

Choice D
Classification: Fractions and Decimals
Snapshot: Dividing by any number is exactly the same as multiplying by that number's reciprocal (and vice-versa).

Dividing 100 by 0.75 is the same as multiplying 100 by the reciprocal of 0.75. The reciprocal of 0.75 is 1.33, not 1.25!

39. Spiral ( )

Choice A
Classification: Fractions and Decimals
Snapshot: This problem highlights reciprocals in the context of a numerical sequence.

The first five terms in this sequence unfold as follows:

$$2 \rightarrow \frac{3}{2} \rightarrow \frac{5}{3} \rightarrow \frac{8}{5} \rightarrow \frac{13}{8}$$

First term: 2

Second term: $1+\frac{1}{2}=\frac{3}{2}$

Third term: $1+\frac{2}{3}=\frac{5}{3}$

Fourth term: $1+\frac{3}{5}=\frac{8}{5}$

Fifth term: $1+\frac{5}{8}=\frac{13}{8}$

40. Discount ( )

Choice D
Classification: Percentage Problem
Snapshot: Percentage problems are easily solved by expressing the original price in terms of 100 percent.

A 10% discount followed by a 30% discount amounts to an overall 37% discount based on original price.

$90\% \times 70\% = 63\%$
$100\% - 63\% = 37\%$

Note that we cannot simply add 10% to 30% to get 40% because we cannot add (or subtract) the percents of different wholes.

41. Inflation ( )

Choice C
Classification: Percentage Problem
Snapshot: This problem highlights the commutative property of multiplication in which *order* doesn't matter.

$120\% \times 110\% = 132\%$
$110\% \times 120\% = 132\%$

It does not matter the *order* in which we multiply numbers, the answer remains the same. In this case, a 20% inflationary increase followed by a 10% inflationary increase is the same as an inflationary increase of 10% followed by an inflationary increase of 20%. Either way we have an overall inflationary increase of 32%.

In contrast with the previous problem titled *Discount,* this problem does not require the actual amount of overall increase but rather the relationship between the two inflationary increases. For the record, a 10% discount followed by a 30% discount is the same as a 30% discount followed by a 10% discount. Test: $0.9 \times 0.7 = 0.63 \times 100\% = 63\%$ and $0.7 \times 0.9 = 0.63 \times 100\% = 63\%$. Either way, we have an overall discount of 37%.

Author's note: The *commutative law* of mathematics states that order doesn't matter. This law holds for addition and multiplication but it does <u>not</u> hold for subtraction or division. Here are some examples.

Addition:	Multiplication:
$a + b = b + a$	$a \times b = b \times a$
$2 + 3 = 3 + 2$	$2 \times 3 = 3 \times 2$

BUT NOT:

Subtraction:

$a - b \neq b - a$
$4 - 2 \neq 2 - 4$

Division:

$a \div b \neq b \div a$
$4 \div 2 \neq 2 \div 4$

42. Gardener ( )

Choice B
Classification: Percentage Problem
Snapshot: This problem introduces percentage increase and decrease problems as they relate to geometry.

View the area of the original rectangular garden as having a width and length of 100%. The new rectangular garden has a length of 140% and a width of 80%. A 20 percent decrease in width translates to a width of 80% of the original width.

Area of original garden:

$$A = lw = (100\% \times 100\%) = 100\%$$

Area of resultant garden:

$$A = lw = (140\% \times 80\%) = 112\%$$

Percent change is as follows:

$$\frac{\text{New} - \text{Old}}{\text{Old}} = \frac{(112\% - 100\%)}{100\%} = \frac{12\%}{100\%} = 12\%$$

Author's note: The calculation below is technically more accurate. The percent signs (i.e., %) cancel out and 0.12 must be multiplied by 100% in order to turn this decimal into a percentage and to reinstate the percentage sign in the final answer.

$$\frac{\text{New} - \text{Old}}{\text{Old}} = \frac{(112\% - 100\%)}{100\%} = \frac{12\%}{100\%} = 0.12 \times 100\% = 12\%$$

As a final note, an even more concise calculation for this problem results from the use of decimals.

$1.4 \times 0.8 = 2.12$
$2.12 - 1.0 = 1.12 \times 100\% = 112\%$
$112\% - 100\% = 12\%$

43. Microbrewery ( )

Choice C
Classification: Percentage Problem
Snapshot: This problem highlights the difference between "percentage increase" and "percentage of an original number."

Note that this problem is in essence asking about productivity: productivity = output ÷ work hours. Use 100% as a base, and add 70% to get 170% and then divide 170% by 80% to get 212.5%. An even simpler calculation involves the use of decimals:

$$\frac{1.7}{0.8} = 2.125 \times 100\% = 212.5\%$$

Now calculate the percent increase:

$$\frac{\text{New} - \text{Old}}{\text{Old}} = \frac{212.5\% - 100\%}{100\%} = \frac{112.5\%}{100\%} = 112.5\%$$

Encore! What if the wording to this problem had been identical except that the last sentence read:

> The year-end factory output per hour is what percent of the beginning of the year factory output per hour?
>
> A) 50%
> B) 90%
> C) 112.5%
> D) 210%
> E) 212.5%

The answer would be choice E. This problem is not asking for a percent increase, but rather "percentage of an original number."

Percentage of an original number:

$$\frac{\text{New}}{\text{Old}} \qquad \frac{212.5\%}{100\%} = 212.5\%$$

Again, the calculation below is more accurate. The percent signs (i.e., %) cancel out and 2.125 must be multiplied by 100% in order to turn this decimal into a percentage and to reinstate the percentage sign in the final answer.

$$\frac{212.5\%}{100\%} = 2.125 \times 100\% = 212.5\%$$

Author's note: The fact that two of the answer choices, namely choices C and E, are 100% apart alerts us to the likelihood that a distinction needs to be made between "percentage increase" and "percentage of an original number."

44. Squaring Off ( )

Choice A
Classification: Percentage Problem
Snapshot: A number of tricky geometry problems can be solved by picking numbers such as 1, 100, or 100 percent.

i) Area of Original Square:

$$\text{Area} = \text{side}^2$$

$$A = s^2$$

$$A = (1)^2 = 1 \text{ square unit}$$

ii) Area of Resultant Square:

$$\text{Area} = \text{side}^2$$

$$A = s^2$$

$$A = (2)^2 = 4 \text{ square units}$$

$$\frac{\text{Original Square}}{\text{Resultant Square}} = \frac{1}{4} = 25\%$$

Author's note: As a matter of form, we generally express a larger number in terms of a smaller number. For example, we tend to say that A is three times the size of B rather than saying that B is one-third the size of A. But it's not technically wrong to express the smaller value in terms of the larger value. Here the resultant square is four times the size of the original square. It is equally correct to say that the smaller, original square figure is one quarter (or 25 percent) the size of the larger, resultant square.

45. Diners ( )

Choice B
Classification: Percentage Problem
Snapshot: This problem highlights a subtle mathematical distinction. In terms of percentages, and increase from 80 percent to 100 percent is not the same as an increase from 100 percent to 120 percent.

Ho, ho—this nice, round number represents the cost before tax (and tip).
Let x be the cost of food before tip:

$$\frac{x}{\$264} = \frac{100\%}{120\%}$$

$$120\%(x) = 100\%(\$264)$$

$$\frac{1}{\cancel{120\%}} \times \cancel{120\%}(x) = \frac{1}{120\%} \times \$264(100\%)$$

$$x = \frac{\$264(100\%)}{120\%}$$

$$x = \frac{\$264(1.0)}{1.2}$$

$$x = \$220$$

Now let x be the cost of food before tip and tax:

$$\frac{x}{\$220} = \frac{100\%}{110\%}$$

$$110\%(x) = 100\%(\$220)$$

$$\frac{1}{\cancel{110\%}} \times \cancel{110\%}(x) = \frac{1}{110\%} \times \$220(100\%)$$

$$x = \frac{\$220(100\%)}{110\%}$$

$$x = \frac{\$220(1.0)}{1.1}$$

$$x = \$200$$

Author's note: The quick method is to divide \$264 by 1.2 and then by 1.1. That is, $[(\$264 \div 1.2) \div 1.1)] = \$200$. Likewise, we can divide \$264 by 1.32. That is, \$264 ÷ (1.2 × 1.1) = \$264 ÷ 1.32 = \$200.

46. Investments ( )

Choice E
Classification: Percentage Problem
Snapshot: You cannot add or subtract the percents of different wholes. "Twenty percent of a big number results in a larger value than 20 percent of a small number."

Below are the calculations for gain and loss expressed as proportions:

Gain on Sale of Property A:

$$\frac{120\%}{100\%} = \frac{\$24{,}000}{x}$$

$$120\%(x) = 100\%(\$24,000)$$

$$\frac{1}{\cancel{120\%}} \times \cancel{120\%}(x) = \frac{1}{120\%} \times 100\%(\$24,000)$$

$$x = \frac{\$24,000(100\%)}{120\%}$$

$$x = \frac{\$24,000(1.0)}{1.2}$$

$$x = \$20,000$$

$$\text{Gain: } \$24,000 - \$20,000 = \$4,000$$

This gain represents the sales price less original purchase price.

Loss on the Sale of Property B:

$$\frac{100\%}{80\%} = \frac{x}{\$24,000}$$

$$100\%(\$24,000) = 80\%(x)$$

$$80\%(x) = 100\%(\$24,000)$$

$$x = \frac{\$24,000(100\%)}{80\%}$$

$$x = \frac{\$24,000(1.0)}{0.8}$$

$$x = \$30,000$$

$$\text{Loss: } \$30,000 - \$24,000 = \$6,000$$

This loss represents the original purchase price less the amount received from the sale.

Therefore, we have an overall loss of $2,000 (net $6,000 loss and $4,000 gain). Note that the following provides shortcut calculations:

Calculation of gain:

$24,000 ÷ 1.2 = $20,000
Gain: $24,000 – $20,000 = $4,000

Calculation of loss:

$24,000 ÷ 0.8 = $30,000
Loss: $30,000 – $24,000 = $6,000

Again, we have an overall loss of $2,000 (net $6,000 loss and $4,000 gain).

47. Earth Speed ()

Choice E
Classification: Ratios and Proportions
Snapshot: Observe how quantities expressed in certain units can be changed to quantities in other units by smartly multiplying by 1.

This problem proves a bit more cumbersome. Here's a three-step approach:

i) Visualize the end result:

$$\frac{20 \text{ miles}}{1 \text{ second}} \times \underline{\qquad} \times \underline{\qquad} = \frac{? \text{ km}}{\text{hour}}$$

ii) Anticipate the canceling of units:

$$\frac{20 \text{ miles}}{1 \text{ second}} \times \frac{\text{seconds}}{\text{hour}} \times \frac{\text{km}}{\text{mile}} = \frac{? \text{ km}}{\text{hour}}$$

iii) Enter conversions and cancel units:

$$\frac{20 \cancel{\text{ miles}}}{1 \cancel{\text{ second}}} \times \frac{3{,}600 \cancel{\text{ seconds}}}{1 \text{ hour}} \times \frac{1 \text{ km}}{0.6 \cancel{\text{ mile}}} = \frac{? \text{ km}}{\text{hour}}$$

Note that 1 hour equals 3,600 seconds and 1 kilometer equals 0.6 miles.

$$\frac{20 \times 3{,}600}{0.6} = \frac{72{,}000}{0.6} = 120{,}000 \text{ kilometer/hour}$$

Note also that 1 hour equals 3,600 seconds; 1 kilometer = 0.6 miles.

48. Rum & Coke ()

Choice B
Classification: Ratios and Proportions
Snapshot: Part-to-part ratios are not the same as part-to-whole ratios. If the ratio of married to non-married people at a party is 1 : 2, the percentage of married persons at the party is one out of three persons or $33\frac{1}{3}$ percent (not 50 percent).

The trap answer is choice A because it erroneously adds component parts of the two different ratios. That is, 1 : 2 + 1 : 3 does not equal 1 + 1 : 2 + 3 = 2 : 5. This could only be correct if ratios represent identical volumes. We cannot simply add two ratios together unless we know the numbers behind the ratios.

	Total	Rum	Coke
Solution #1	6	2	4
Solution #2	32	8	24
Totals		10	28
		Simplify☺	
Final Ratio		5	14

The ratio of 10 : 28 simplifies to 5 : 14.

Supporting Calculations:

$6 \times \frac{1}{3} = 2$ Two ounces of rum in solution #1

$6 \times \frac{2}{3} = 4$ Four ounces of Coke in solution #1

$32 \times \frac{1}{4} = 8$ Eight ounces of rum in solution #2

$32 \times \frac{3}{4} = 24$ Twenty-four ounces of Coke in solution #2

49. Millionaire ( )

Choice A
Classification: Ratios and Proportions
Snapshot: Triple ratios (3 parts) are formed by making the middle term of equivalent size.

Correct answers would include any and all multiples of 1 : 4 : 400, including 2 : 8 : 800, 4 : 16 : 1600, etc. However, the latter choices are not presented as options here.

i) Visualize the Solution

Billionaire	Millionaire	Yuppie
\$20 ←- - - - - - - - - - - - →	\$0.20	
	\$4 ←- - - - - - - - - - - →	\$1

ii) Do the Math

Billionaire	Millionaire	Millionaire	Yuppie	
\$20	\$0.20	\$4	\$1	Original Ratio
×20	×20	×1	×1	Adjusting Multipliers
\$400	\$4	\$4	\$1	Resultant Ratio
	Equivalent			

Choose the Answer:

B to M to Y $\rightarrow$ \$400 to \$4 to \$1
$\therefore$ Y to M to B $\rightarrow$ \$1 to \$4 to \$400

Author's note: Triple ratios (e.g., A : B : C) are formed from two pairs of ratios by making sure the "middle terms" are of equivalent size.

50. Deluxe ( )

Choice D
Classification: Ratios and Proportions
Snapshot: Deluxe is considered a difficult ratio problem. The first step is to break the 24 liters of fuchsia into "red" and "blue." This requires using a part-to-whole ratio (i.e., three-eighths blue and five-eighths red; $\frac{3}{8}$ and $\frac{5}{8}$ respectively). Our final ratio is a part-to-part ratio, comparing red and blue paint in fuchsia to the red and blue paint in mauve.

First we know that there are 24 liters of fuchsia in a ratio of 5 parts red to 3 parts blue. We break down this amount into the actual amount of red and blue in 24 liters of fuchsia.

Blue: 5 parts red to 3 parts blue.

$$\frac{3}{5+3}=\frac{3}{8} \rightarrow \frac{3}{8}\times 24 = 9 \text{ liters of blue paint}$$

Red: 5 parts red to 3 parts blue.

$$\frac{5}{5+3}=\frac{5}{8} \rightarrow \frac{5}{8}\times 24 = 15 \text{ liters of red paint}$$

So the final formula, expressed as a proportion, becomes:

$$\frac{15 \text{ red}}{9 \text{ blue} + x \text{ blue}} = \frac{3 \text{ red}}{5 \text{ blue}}$$

$5(15)=3(9+x)$

$75=3(9+x)$

$75 = 27 + 3x$

$3x+27=75$

$3x=48$

$x=16$ liters of blue paint

51. Rare Coins ( )

Choice E
Classification: Ratios and Proportions
Snapshot: This problem highlights two different problem solving approaches for ratio type problems: the "two-variable, two-equations approach" and the "multiples approach."

There are two ways to solve this problem algebraically. The first approach is to use the two-variable, two-equations approach. The second approach is to use the multiples approach.

I. Two-Variable, Two-Equations Approach

Using this approach, we identify two equations and substitute one variable for another. G represents gold coins; S represents silver coins.

First equation:

$$\frac{1}{3} = \frac{G}{S} \quad or \quad S = 3G$$

Second equation:

$$\frac{G+10}{S} = \frac{1}{2} \quad or \quad S = 2(G+10)$$

Since $S = 3G$ and $S = 2(G+10)$, we can substitute for one of these variables and solve for the other.

$2(G+10) = 3G$

$2G + 20 = 3G$

$G = 20$ and, therefore, $S = 60$

Per above, we substitute $G = 20$ into either of the two original equations and obtain $S = 60$.

Finally, 20 (gold coins) *plus* 60 (silver coins) *plus* 10 (gold coins added) *equals* 90 total coins.

II. Multiples Approach

The secret behind this approach is to view x as representing multiples of the actual number of coins. Given a ratio of 1 to 3, we can represent the actual number of gold coins versus non-gold coins as $1x$ and $3x$ respectively. The solution is as follows:

$$\frac{1x+10}{3x} = \frac{1}{2}$$

$2(1x+10) = 3x$

$2x + 20 = 3x$

$x = 20$

Substituting 20 for x in the original equals:

$$\frac{\text{coins (gold)}}{\text{coins (silver)}} = \frac{1x}{3x} = \frac{1(20)}{3(20)} = \frac{20}{60}$$

Thus, 20 (gold coins) *plus* 60 (silver coins) *plus* 10 (gold coins added) *equals* 90 total coins.

Author's note: In the event of guessing, since the final ratio is 2 to 1, this means that the total number of coins must be a multiple of 3. Only answer choices C (60) or E (90) could therefore be correct.

52. Coins Revisited ( )

Choice B
Classification: Ratios and Proportions
Snapshot: *Coins Revisited* differs from the problem *Rare Coins* in that the total number of coins in the collection (per *Coins Revisited*) does not change. Mathematically, 10 coins are subtracted from the denominator while coins 10 coins are added to the numerator. In this particular problem, the second statement, "If 10 more gold coins were to be subsequently traded…," is treated mathematically no different than if an actual trade had occurred.

The secret to this particular problem lies in first translating the part-to-whole ratio of 1 to 6 to a part-to-part ratio of 1 to 5.

I. Two-Variable, Two-Equations Approach

First equation:

$$\frac{1}{5} = \frac{G}{S} \qquad S = 5G$$

Second equation:

$$\frac{G+10}{S-10} = \frac{1}{4} \qquad 4(G+10) = 1(S-10)$$

Since $S = 5G$ and $4(G+10) = 1(S-10)$ we can substitute and solve for G (or S).

$4(G+10) = 1(5G-10)$
$4G + 40 = 5G - 10$
$-G = -50$
$(-1)(-G) = (-1)(-50)$
$G = 50$ and, therefore, $S = 250$

Thus, there are 60 gold coins after the trade (i.e., 50 gold coins plus 10 gold coins added).

II. Multiples Approach

Here $1x$ and $5x$ can be viewed as representing multiples of the actual number of gold coins and silver coins, respectively. The solution is as follows:

$$\frac{1x+10}{5x-10}=\frac{1}{4}$$

$$4(1x+10)=1(5x-10)$$

$$4x+40=5x-10$$

$$x=50$$

Substituting 50 for x in the original equals:

$$\frac{\text{coins (gold)}}{\text{coins (non-gold)}}=\frac{1x}{5x}=\frac{1(50)}{5(50)}=\frac{50}{250}$$

50 (gold coins) *plus* 10 gold coins added equals 60 coins.

Author's note: For the record, there are 300 gold coins in the collection (both before and after the proposed trade). Before the trade, there are 50 gold coins and 250 silver coins. After the trade, there would be 60 gold coins and 240 silver coins.

53. Plus-Zero ( )

Choice D
Classification: Squares and Cubes
Snapshot: First, consider which of the seven numbers—(i.e., 2, −2, 1, −1, $\frac{1}{2}$, $-\frac{1}{2}$, and 0)—satisfy each of the conditions presented in statements I through III. When a problem states $x > 0$, three numbers should immediately come to mind: 2, 1, and $\frac{1}{2}$.

Statement I:

Could x^3 be greater than x^2?
Answer—yes.

Example $2^3 > 2^2$
Proof $8 > 4$

Statement II:

Could x^2 be equal to x?
Answer—yes.

Example $1^2 = 1$
Proof $1 = 1$

Statement III:

Could x^2 be greater than x^3?
Answer—yes.

Example $\left(\frac{1}{2}\right)^2 > \left(\frac{1}{2}\right)^3$

Proof $\frac{1}{4} > \frac{1}{8}$

54. Sub-Zero ( )

Choice B
Classification: Squares and Cubes

Snapshot: When a problem states $x < 0$, three numbers should immediately come to mind: –2, –1, and $-\frac{1}{2}$.

Statement I:

Is x^2 greater than 0?
Answer—absolutely. As long as x is negative, it will, when squared, become positive.

Statement II:

Is $x - 2x$ greater than 0?
Answer—absolutely. As long as x is negative the expression "$x - 2x$" will be greater than zero.

Statement III:

Is $x^3 + x^2$ less than 0?
Answer—not necessarily.

Example $\left(-\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^2$

$$\left(-\frac{1}{8}\right) + \frac{1}{4} = \frac{1}{8}$$

$$\frac{1}{8} > 0$$

55. Solar Power ( )

Choice B
Classification: Exponent Problem
Snapshot: This problem tests the ability to manipulate exponents.

$$\frac{2 \times 10^{30}}{8 \times 10^{12}} = 0.25 \times 10^{18} = 2.5 \times 10^{17}$$

Note that by moving the decimal one place to the right (i.e., 0.25 to 2.5), we reduce the power of the exponent by one (i.e., 10^{18} becomes 10^{17}).

56. Bacteria ( )

Choice C
Classification: Exponent Problem
Snapshot: This problem shows the multiplicative power of numbers.

Visualize the solution. We start with $(10)^5$ then multiply by 2 for each 10-minute segment. Since there are six 10-minute segments in one hour, we arrive at $[2 \times 2 \times 2 \times 2 \times 2 \times 2 \times (10^5)]$. Thus, $(2^6)(10^5)$ represents the number of bacteria after one hour.

57. K.I.S.S. ( )

Choice E
Classification: Exponent Problem
Snapshot: Picking numbers may be used as an alternative approach in solving exponent problems.

I. Algebraic Method

$3^a + 3^{a+1}$
$3^a + 3^a \times 3^1$
$3^a(1 + 3^1)$ [factor out 3^a from both terms]
$3^a(1 + 3)$
$3^a(4)$ or $4(3^a)$ [the use of brackets here is simply a matter of form]

II. Picking Numbers Method

Another method that can be used to solve this problem is substitution, which involves picking numbers. Take the original expression: $3^a + 3^{a+1}$. Substitute the "easiest integer." That is, let's substitute $a = 1$. Therefore, $3^1 + 3^{1+1} = 3^1 + 3^2 = 3 + 9 = 12$. We ask ourselves: "Which answer choice gives us 12 when we substitute $a = 1$ into that equation?" Answer: Choice E.

Proof: $4(3^a) = 4(3^1) = 12$.

58. Triplets ( )

Choice A
Classification: Exponent Problem
Snapshot: Consistent with Exponent Rule 8 (see page 31), we can simplify this expression by factoring out a common term (i.e., 3^{10}).

$3^{10} + 3^{10} + 3^{10}$
$3^{10}(1 + 1 + 1)$
$3^{10}(3)$
$3^{10} \times 3^{1} = 3^{11}$

Author's note: Here's a bonus question:

$$\frac{2^{15} - 2^{14}}{2} = ?$$

A) 1 B) 2 C 2^{7} D) 2^{13} E) 2^{14}

Calculation:

$$\frac{2^{15} - 2^{14}}{2} = \frac{2^{14}(2^{1} - 1)}{2} = \frac{2^{14}(2 - 1)}{2} = \frac{2^{14}(1)}{2} = \frac{2^{14}}{2^{1}} = 2^{13}$$

Choice D is therefore correct.

59. The Power of 5 ( )

Choice C
Classification: Exponent Problem
Snapshot: This problem highlights the multiplying of exponents consistent with Exponent Rule 3—"power of a power" (see page 30).

$5^{5} \times 5^{7} = (125)^{x}$

$5^{12} = (125)^{x}$ Per Exponent Rule 1

$5^{12} = (5^{3})^{x}$ Per Exponent Rule 3

$5^{12} = (5^{3})^{4}$ Therefore, $x = 4$

$5^{12} = 5^{12}$

Note that in the penultimate step above, given that the bases are equal in value, the exponents must also be equal in value. That is, in terms of exponents, $12 = 3x$ and $x = 4$. Here's a somewhat easier scenario:

$10^3 \times 10^5 = (100)^x$

$10^8 = (100)^x$ Per Exponent Rule 1

$10^8 = (10^2)^x$ Per Exponent Rule 3

$10^8 = (10^2)^4$ Therefore, $x = 4$

$10^8 = 10^8$

Note that with respect to exponents, $8 = 2x$ and $x = 4$.

60. M&N ()

Choice B
Classification: Exponent Problem
Snapshot: This problem highlights a more difficult exponent problem containing two variables. It also highlights Exponent Rule 4 (see page 30).

There are two ways to solve this problem. The first is the *algebraic method* and the second is the *picking numbers* method.

I. Algebraic Method

$n = 2^{m-1}$

$n = 2^m \times 2^{-1} = \dfrac{2^m}{2^1}$

$2n = 2^m$

$4^m = (2 \times 2)^m = 2^m \times 2^m$ (Per Exponent Rule 4)

$4^m = 2n \times 2n = 4n^2$ (Note: $2n = 2^m$)

Note that in the penultimate step above, $4^m = 2^m \times 2^m$, and since $2^m = 2n$, the final calculation becomes $2n \times 2n$.

II. Picking Numbers Method

Since $m > 1$, pick $m = 2$, such that $n = 2^{m-1} = 2^{2-1} = 2^1 = 2$.

When $m = 2$ it is also true that $4^m = 4^2 = 16$.

So the question becomes: When $m = 2$ which answer, A thru E, when substituting $n = 2$, will result in an answer of 16.

Choice B is correct: $4n^2 = 4(2)^2 = 16$.

61. Incognito ( )

Choice E
Classification: Exponent Problem
Snapshot: This problem shows how fractions can be simplified through factoring. The spotlight is on Exponent Rule 8 (see page 31).

The key is to first factor out "$(2^2)(3^2)$" from each of the denominators. This treatment is consistent with Exponent Rule 8.

A) $\dfrac{25}{(2^4)(3^3)} \rightarrow \dfrac{25}{(2^2)(3^1)} = \dfrac{25}{12} = 2\dfrac{1}{12}$

B) $\dfrac{5}{(2^2)(3^3)} \rightarrow \dfrac{5}{(1)(3^1)} = \dfrac{5}{3} = 1\dfrac{2}{3}$

C) $\dfrac{4}{(2^3)(3^2)} \rightarrow \dfrac{4}{(2^1)(1)} = \dfrac{4}{2} = 2$

D) $\dfrac{36}{(2^3)(3^4)} \rightarrow \dfrac{36}{(2^1)(3^2)} = \dfrac{36}{18} = 2$

E) $\dfrac{76}{(2^4)(3^4)} \rightarrow \dfrac{76}{(2^2)(3^2)} = \dfrac{76}{36} = 2\dfrac{4}{36} = 2\dfrac{1}{9}$

Author's note: One theory in terms of guessing on GMAT math problems relates to "Which of the following" questions (also known as "WOTF" math questions). In this question type, test makers tend to manifest answers deep in the answer choices, meaning that choices D and E have a disproportional chance of ending up as correct answers. Why is this? "Which of the following" questions require the test taker to work with the answer choices, and most candidates logically work from choices A to E. This presents two opportunities. If we need to guess on these questions, it is best to guess choices D or E. Also, it is judicious to start checking answers in reverse order, starting with choice E.

62. Chain Reaction ( )

Choice D
Classification: Exponent Problem
Snapshot: This follow-up problem is a more difficult problem than the preceding one but the suggested approach is identical.

If, $x - \dfrac{1}{2^6} - \dfrac{1}{2^7} - \dfrac{1}{2^8} = \dfrac{2}{2^9}$ then $x = \dfrac{2}{2^9} + \dfrac{1}{2^8} + \dfrac{1}{2^7} + \dfrac{1}{2^6}$

Now factor out $\dfrac{1}{2^6}$ from each of the terms in the denominator:

$$\text{So } x = \frac{1}{2^6}\left(\frac{2}{2^3}+\frac{1}{2^2}+\frac{1}{2^1}+1\right)$$

$$= \frac{1}{2^6}\left(\frac{2}{8}+\frac{1}{4}+\frac{1}{2}+1\right)$$

$$= \frac{1}{2^6}\left(1+1\right) = \frac{1}{2^6}(2) = \frac{1}{2^5}$$

63. Simplify ( )

Choice A
Classification: Radical Problem
Snapshot: This problem illustrates how to simplify radicals and brings into play Radical Rule 7 (see page 33).

$$\sqrt{\frac{(12\times 3)+(4\times 16)}{6}} = \sqrt{\frac{36+64}{6}} = \sqrt{\frac{100}{6}} = \sqrt{\frac{50}{3}}$$

$$\frac{\sqrt{50}}{\sqrt{3}} = \frac{\sqrt{25\times 2}}{\sqrt{3}} = \frac{\sqrt{25}\times\sqrt{2}}{\sqrt{3}} = \frac{5\sqrt{2}}{\sqrt{3}}$$

$$\frac{5\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{2\times 3}}{\sqrt{9}} = \frac{5\sqrt{6}}{3}$$

Note that you cannot break up the radical at the addition sign into two parts:

$$\neq \frac{\sqrt{12\times 3}+\sqrt{4\times 16}}{6} = \frac{\sqrt{36}+\sqrt{64}}{6} = \frac{6+8}{6} = \frac{14}{6} = 2\frac{1}{3}$$

This result is incorrect!

64. Tenfold ( )

Choice C
Classification: Radical Problem
Snapshot: This problem highlights Radical Rule 3 (see page 32).

$$\frac{\sqrt{10}}{\sqrt{0.001}} = \sqrt{\frac{10}{0.001}} = \sqrt{10{,}000} = 100$$

65. Strange ( )

Choice A
Classification: Radical Problem
Snapshot: This problem illustrates how the “multiplicative inverse” can be used to simplify radical equations; it illustrates Radical Rule 8 (see page 33).

The solutions approach is to multiply the denominator of the fraction by its multiplicative inverse.

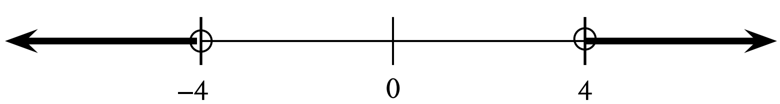
$$\left(\frac{1-\sqrt{2}}{1+\sqrt{2}}\right)\times\left(\frac{1-\sqrt{2}}{1-\sqrt{2}}\right)=\frac{1-\sqrt{2}-\sqrt{2}+\sqrt{4}}{1-\sqrt{2}+\sqrt{2}-\sqrt{4}}=\frac{1-2\sqrt{2}+\sqrt{4}}{1-\sqrt{4}}=\frac{1-2\sqrt{2}+2}{1-2}=\frac{3-2\sqrt{2}}{-1}$$

$$\frac{3-2\sqrt{2}}{-1}=\frac{3-2\sqrt{2}}{-1}\times\frac{-1}{-1}=\frac{-3+2\sqrt{2}}{+1}=-3+2\sqrt{2}$$

66. Two-Way Split ( )

Choice A
Classification: Inequality Problem
Snapshot: This problem tests our ability to express an inequality solution in a single solution, with a single variable x. Also, when multiplying through by a negative number, we reverse the direction of the inequality sign.

Note that another correct answer could have been expressed as following: $x < -4$ or $x > 4$, which is an alternative way of writing the former expression.



$-x^2 + 16 < 0$

$(-1)(x^2) + 16 < 0$

$(-1)(-1)(x^2) + (-1)16 > (-1)0$ [multiply each term through by –1]

$x^2 - 16 > 0$

Therefore, $x < -4$ or $x > 4$. When combining these two inequalities into one expression, we write it as: $-4 > x > 4$. Note that in the penultimate step of our calculation above, we multiply each term of the equation by –1 in order to cancel the negative sign in front of x. In multiplying through by –1 we must remember to reverse the inequality sign.

67. Primed ( )

Choice C
Classification: Prime Number Problem
Snapshot: To review prime numbers and prime factorization.

"x"	Factors	Prime Factors	"Primeness"
A) 10	1, 2, 5, 10	2, 5	$5 - 2 = 3$
B) 12	1, 2, 3, 4, 6, 12	2, 3	$3 - 2 = 1$
C) 14	1, 2, 7, 14	2, 7	$7 - 2 = 5$
D) 15	1, 3, 5, 15	3, 5	$5 - 3 = 2$
E) 18	1, 2, 3, 6, 9, 18	2, 3	$3 - 2 = 1$

68. Odd Man Out ( )

Choice C
Classification: Prime Number Problem
Snapshot: This problem helps reveal the mathematical reason for why one number is or is not a multiple of another number.

First let's visualize P as: $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 \times 13$
This is also the equivalent of 13! (or 13 factorial)

Statement I is false. P is an even number. As long as we have a least one even number in our multiplication sequence, the entire product will be even. Remember an even number times an odd number or an even number times an even number is always an even number. For the record, P actually equals 6,227,020,800.

Statement II is false; statement III is true. The key here is to look at the prime factors of P. These include: 2, 3, 5, 7, 11, and 13. For P to be a multiple of any number, that number must not contain any prime number that isn't already contained in P. What are the distinct prime factors of 17?

Factorization	Prime Factors	Distinct Prime Factors
$17 = 1 \times 17$	17	17
$24 = 1 \times 24$		
$24 = 1 \times 8 \times 3$		
$24 = 1 \times 2 \times 2 \times 2 \times 3$	2, 2, 2, 3	2, 3

The number 17 has as its (only) prime factor, the number 17. Since P does not contain this number it will not be a multiple of 17. It's as simple as that. Think of prime factors as the "DNA" of numbers. Any number x will not be a multiple of y if y contains any distinct prime number not included in x. Stated another way, x will be a multiple of y only if x contains, at a minimum, the same number of distinct prime factors as y does. For example, the number 6 is a multiple of 3 because 3 contains no prime numbers that aren't already included in 6 (i.e., 2, 3). The number 6 is not a multiple of 5 because 5 has a prime factor 5 which is not shared with the prime factors of the number 6.

P is a multiple of 24 because P contains all of the distinct prime numbers that 24 has.

69. Remainder ( )

Choice E
Classification: Remainder Problem
Snapshot: To review how to pick numbers for use in solving multiple and remainder problems.

A key step in this problem involves picking a number for k to work with based on the original information that k when divided by 7 leaves a remainder 5. This number is 12. We now substitute 12 for k.

I.	$4k+7$	$4k+7=4(12)+7=55$ $55 \div 7 = 7,$ with a remainder of 6
II.	$6k+4$	$6k+4=6(12)+4=76$ $76 \div 7 = 10,$ with a remainder of 6
III.	$8k+1$	$8k+1=8(12)+1=97$ $97 \div 7 = 13,$ with a remainder of 6

70. Double Digits ( )

Choice D
Classification: Remainder Problem
Snapshot: When faced with multi-step divisibility problems (A divided by B leaves x but A divided by C leaves y), find only those numbers which satisfy the first scenario then use this short-list of numbers to determine the solution to the next scenario.

The key to this problem is to do one part at a time rather than trying to combine the information. For example, list all two-digit numbers that when divided by 10 leave 3. These numbers include: 13, 23, 33, 43, 53, 63, 73, 83, and 93. Next, examine these numbers and determine which of these, when divided by 4, will leave a remainder of 3. These numbers include: 23, 43, 63, and 83.

Numbers	Reminder
13	1
23	3⇦
33	1
43	3⇦
53	1
63	3⇦
73	1
83	3⇦
93	1

71. Visualize ( )

Choice A
Classification: Symbolism Problem
Snapshot: Learning to visualize the solution is the key to conquering symbolic or "make-believe" operations.

Set the problem up conceptually by first visualizing the solution:

$$V^* = V - \frac{V}{2}$$

$$(V^*)^* = V - \frac{V}{2} - \left(\frac{V - \frac{V}{2}}{2}\right)$$

$$3 = V - \frac{V}{2} - \left(\frac{V - \frac{V}{2}}{2}\right)$$

Calculate the outcome algebraically:

Multiply each term in the equation by 2.

$$(2)3 = (2)V - (\cancel{2})\frac{V}{\cancel{2}} - (\cancel{2})\left(\frac{V - \frac{V}{2}}{\cancel{2}}\right)$$

$$6 = 2V - V - \left(V - \frac{V}{2}\right)$$

$$6 = 2V - V - V + \frac{V}{2}$$

Once again, multiply each term through by 2.

$$(2)6 = (2)2V - (2)V - (2)V + (\cancel{2})\frac{V}{\cancel{2}}$$
$$12 = 4V - 2V - 2V + V$$
$$V = 12$$

72. Masquerade ( )

Choice D
Classification: Coordinate Geometry Problem
Snapshot: Positive lines slope upward ("forward slashes"); negative lines slope backward ("back slashes"). Graphs with slopes less than one (positive or negative fractions) are flat and closer to the x-axis. Graphs with slopes greater than one (coefficients > 1) are more upright and closer to the y-axis.

Compare the general slope formula, $y = mx + b$, to the equation at hand: $y = -2x + 2$. For the general slope formula, m is equal to the slope and b is equal to the y-intercept. Therefore, in the equation at hand, the slope is –2. A negative slope tells us that the graph is moving northwest-southeast; a slope of negative 2 tells us that the graph drops two units for every one unit it runs. The y-intercept is 2. This means that one point is (0,2).

Lines A and B are out because they have positive slopes and we are looking for a negative slope. We are looking for a y-intercept of 2 so Line E (choice E) is out. Focus on Lines C and D. Since the slope is –2, this means it drops two units for every one unit it runs and, because it is negative, it is moving northwest-southeast (note that if it had a positive slope, the graph would move southwest-northeast). You may be able to pick out Line D as the immediate winner. If not, test both the y-intercept and x-intercept to be absolutely sure. To test the y-intercept, which we already can see is (0,2), we set x equal to zero: $y = -2(0) + 2$, and 2 is our answer. Line D intersects the y-axis at (0,2) as anticipated. To test the x-intercept, we set y equal to zero: $0 = -2x + 2$, and 1 is our answer. Line D intersects the x-axis at (1,0). Thus, equation Line D is the clear winner based on its slope and its y-intercept and x-intercept.

Author's note: Need additional proof? Whenever you know two points on a line, you can figure out the slope. Using the two points above, (0,2) and (1,0), we can find the slope of our line. Slope equals rise over run or algebraically (it doesn't matter which point is subtracted from the other):

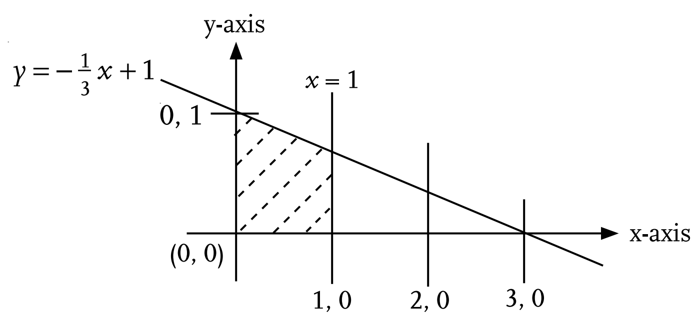
$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{rise}_2 - \text{rise}_1}{\text{run}_2 - \text{run}_1} = \frac{2-0}{0-1} = \frac{2}{-1} = -2$$

Is –2 the slope that we are looking for? Yes.

73. Boxed In ( )

Choice D
Classification: Coordinate Geometry Problem
Snapshot: This problem highlights basic information regarding coordinate geometry. The slope of a horizontal line is $y = (\ldots, -2, -1, 0, 1, 2, \ldots)$. The slope of a vertical line is $x = (\ldots, -2, -1, 0, 1, 2, \ldots)$. The slope of the y-axis is $x = 0$; the slope of the x-axis is $y = 0$.

Go through each answer choice. Choices A, B, C, and E represent boundary lines. Choice A, $x = 0$, is the y-axis. Choice B, $y = 0$, is the x-axis. Choice C, $x = 1$, forms the right boundary; the formula, $y = -\frac{1}{3}x + 1$, forms the top boundary. To test the inappropriateness of choice D, $x - 3y = 0$, try placing various points into the equation. Any set of points on the line should be able to satisfy the equation. For example, take (3,0). Now substitute $x = 3$ and $y = 0$ into the equation in choice D, $x - 3y = 0$. You get $3 - 3(0) = 0$. This doesn't make sense and cannot be the correct equation. Try choice E, $y + \frac{1}{3}x = 1$. Substitute (3,0) and we get $0 + \frac{1}{3}(3) = 1$. This works—it is the proper equation and forms the roof, or top line, of the marked area.



74. Intercept ( )

Choice A
Classification: Coordinate Geometry Problem
Snapshot: The slope formula is $y = mx + b$ where m is defined as the slope or gradient and b is defined as the y-intercept.

Start by visualizing the slope formula: $y = mx + b$. Let's determine the slope first. Slope m equals "rise over run."

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{rise}_2 - \text{rise}_1}{\text{run}_2 - \text{run}_1} = \frac{3-(-5)}{10-(-6)} = \frac{8}{16} = \frac{1}{2}$$

The slope formula now reads: $y = \frac{1}{2}x + b$. To find b let's put in the coordinates of the first point: $3 = \frac{1}{2}(10) + b$; $b = -2$. The complete slope formula becomes: $y = \frac{1}{2}x - 2$.

To find the x-intercept, we set $y = 0$.

$$y = \frac{1}{2}x - 2$$

$$0 = \frac{1}{2}x - 2$$

$$-\frac{1}{2}x = -2$$

$$x = 4$$

75. Magic ( )

Choice A
Classification: Plane Geometry Problem
Snapshot: This problem tests the simple definition of π.

Circumference $= \pi \times$ diameter. Since $C = \pi d$, the ratio of a circle's circumference to its diameter is: $\frac{\pi \cancel{d}}{\cancel{d}} = \pi$. This is the very definition of Pi; Pi is the ratio of the circumference of a circle to its diameter. The circumference of a circle is uniquely $\cong$3.14 times as big as its diameter. This is always true. Choice D cannot be correct. A ratio is a ratio and, as such, does not vary with the size of the circle. For the record, the fractional equivalent of Pi is $\frac{22}{7}$.

76. Kitty Corner ( )

Choice B
Classification: Plane Geometry Problem
Snapshot: To review right-isosceles triangles and the relationships between the relative lengths of their sides, namely: $1:1:\sqrt{2}$.

View the triangular wedge. The height is 2, the base is $2\sqrt{2}$ and the hypotenuse is x. Using the Pythagorean Theorem:

$$a^2 + b^2 = c^2$$

$$(2)^2 + (2\sqrt{2})^2 = (x)^2$$

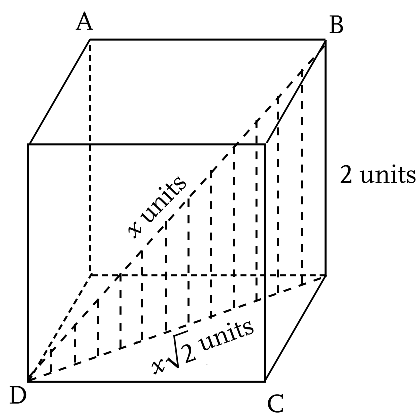
$$4 + 8 = x^2$$

$$x^2 = 12$$

$$x = \sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$$

You may wonder, "How do we know the base is $2\sqrt{2}$." The base of the triangle is really the hypotenuse of the right isosceles triangle which is at the very bottom of the cube. Because all sides of the cube are

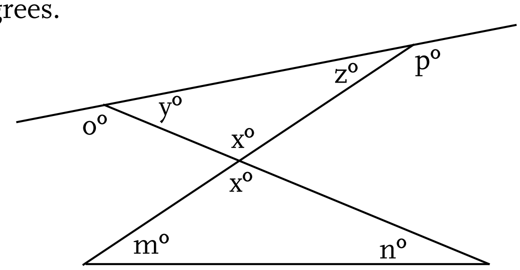
2 units in length, the hypotenuse of the bottom triangle is $2\sqrt{2}$. This information is critical to finding the hypotenuse of the triangle represented by x.



77. Lopsided ( )

Choice C
Classification: Plane Geometry Problem
Snapshot: To illustrate how to find the measures of angles indirectly.

Since $m + n$ =110 degrees; thus $x = 70$ degrees because a triangle n-m-x equals 180. Also triangle y-x-z also measures 180 degrees. The measure of $o + p$ is found by setting the measures of y-x-z equal to 180 degrees. Thus, $x = 70$; $y = 180 - o$; $z = 180 - p$. Finally, $180 = 70 + (180 - o) + (180 - p)$ and $o + p = 250$ degrees.



Note: Figure not drawn to scale

78. Diamond ( )

Choice A
Classification: Plane Geometry Problem
Snapshot: This problem tests an understanding of right-isosceles triangles and the ability to calculate the length of a single side when given the hypotenuse.

Either of the two dotted lines within the square serves to divide the square into two right-isosceles triangles. Since each dotted line has a length of 3 units, each side therefore has a length of $\frac{3}{\sqrt{2}}$. This calculation can be a bit tricky. Given that the ratios of the lengths of the sides of a right-isosceles triangle are $1:1:\sqrt{2}$, we can use a ratio and proportion to calculate the length of the hypotenuse (the dotted line in the diagram that follows):

Standard ratio: $1:1:\sqrt{2}$

Per this problem: $x:x:3$

Ratio solved: $\frac{3}{\sqrt{2}}:\frac{3}{\sqrt{2}}:3$

Calculation:

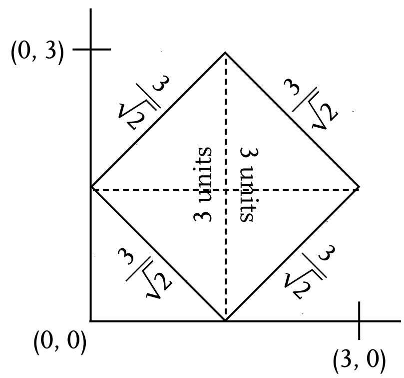
$$\frac{1}{\sqrt{2}}::\frac{x}{3}$$

$$1(3)=x(\sqrt{2})$$

$$\frac{1}{\sqrt{2}}\times 3=\frac{1}{\sqrt{2}}\times x(\sqrt{2})$$

$$\frac{3}{\sqrt{2}}=\frac{x(\cancel{\sqrt{2}})}{\cancel{\sqrt{2}}}$$

$$x=\frac{3}{\sqrt{2}}$$



Calculating Perimeter:

$$P=4s$$

$$P=4\times\frac{3}{\sqrt{2}}=\frac{12}{\sqrt{2}}\text{ units}$$

We typically simplify radicals in order to eliminate having a radical in the denominator of a fraction. This dovetails with Radical Rule 3 (see page 32).

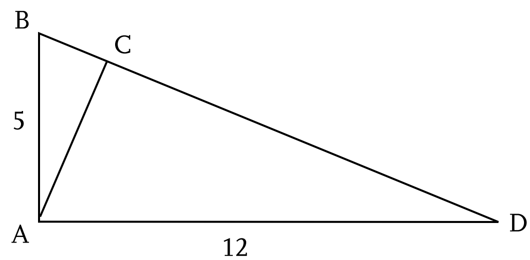
$$\frac{12}{\sqrt{2}}=\frac{12}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{12\sqrt{2}}{\sqrt{4}}=\frac{12\sqrt{2}}{2}=6\sqrt{2}\text{ units}$$

79. AC ( )

Choice E
Classification: Plane Geometry Problem
Snapshot: This problem merely requires that the candidate can calculate the height of a triangle but doing so requires viewing the triangle from different perspectives.

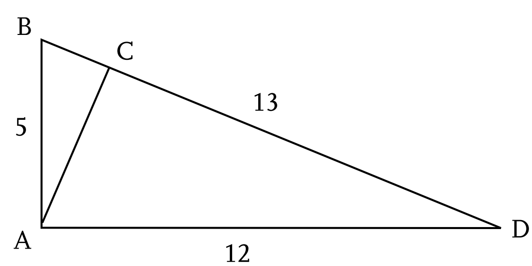
In the original diagram, the measure of BD is easy to calculate using the Pythagorean formula:

$$c^2 = a^2 + b^2$$
$$c^2 = (12)^2 + (5)^2$$
$$c^2 = 144 + 25$$
$$c^2 = 169$$
$$c^2 = \sqrt{169}$$
$$c = 13$$



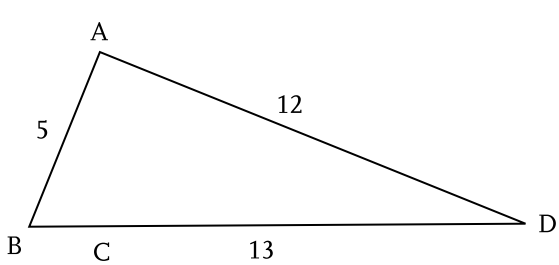
Therefore the measure of BD is 13. As seen in the diagram below, we now know the measures of all sides of the triangle. We can also calculate the area of the triangle:

$$\text{Area} = \frac{bh}{2} = \frac{12 \times 5}{2} = 30 \text{ units}^2$$



When the diagram is flipped it is easy to calculate the height (AC) given that the area is 30 square units and the base is 13 units.

$$A = \frac{bh}{2}$$
$$30 = \frac{bh}{2}$$
$$30 = \frac{13 \times h}{2}$$
$$60 = 13 \times h$$
$$h = \frac{60}{13} \text{ units}$$



80. Circuit ( )

Choice B
Classification: Plane Geometry Problem
Snapshot: This problem introduces geometry problems where the solution is expressed in algebraic terms. It also requires that we remain flexible and be able to work with variables expressed as capital letters as well as lower case letters. Two formulas are needed to solve this problem:

$$A = l \times w \text{ and } P = 2l + 2w$$

Of course, cosmetically speaking, the same as writing each variable with a capital letter:

$$A = L \times W \text{ and } P = 2L + 2W$$

(where A is area, L is length, W is width, and P is perimeter of a given rectangle)

Since none of the answer choices have reference to the variable of length, we substitute for the variable L as follows:

$$P = 2L + 2W \qquad \text{and} \qquad L = \frac{A}{W}$$

$$\text{so } P = 2\left(\frac{A}{W}\right) + 2W$$

Multiplying each term of the equation by W:

$$(W)P = (\cancel{W})2\left(\frac{A}{\cancel{W}}\right) + (W)2W$$

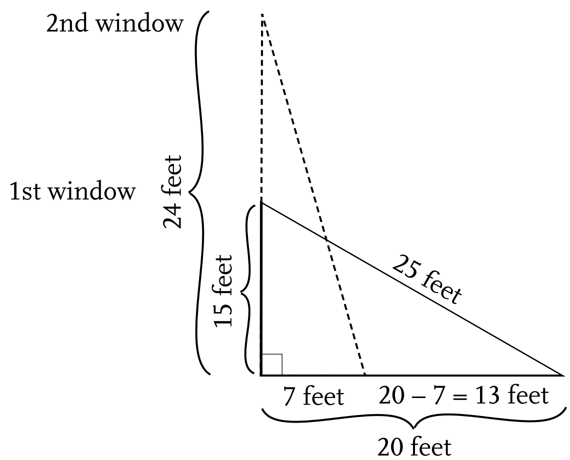
$$PW = 2A + 2W^2$$

$$2W^2 - PW + 2A = 0$$

81. Victorian ( )

Choice D
Classification: Plane Geometry Problem
Snapshot: The Pythagorean Theorem, $a^2 + b^2 = c^2$, can always be used to find the length of the sides of any right triangle. "Pythagorean triplets" are integers which satisfy the Pythagorean Theorem. The four common Pythagorean triplets that appear on the GMAT include: 3:4:5; 5:12:13; 8:15:17; and 7:24:25.

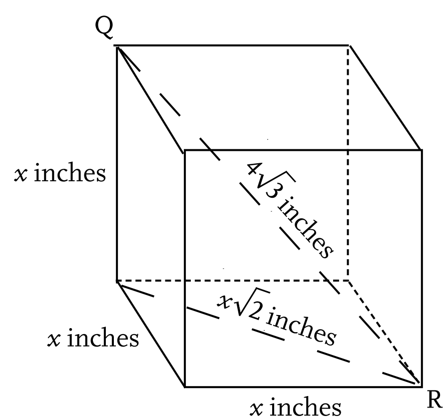
This is a classic problem that can be solved using the Pythagorean Theorem and formula: $a^2 + b^2 = c^2$, where a, b, and c are sides of a triangle and c is the hypotenuse. In this problem, we concentrate on the first window and find the distance from the base of the house as follows: $(15)^2 + (x)^2 = (25)^2$ so $x = 20$. Then we concentrate on the second window and find the distance from the base of the house as follows: $(24)^2 + (x)^2 = (25)^2$ so $x = 7$. Don't forget that the ladder has been moved closer by 13 feet, not 7 feet.



82. QR ( )

Choice C
Classification: Plane Geometry Problem
Snapshot: This more difficult problem works in reverse of the previous one. Whereas the previous problem gave the measure of a side and asked us to calculate the *longest diagonal,* this problem gives the measure the longest diagonal and asks us find the measure of a side, en route to finding volume.

$$a^2 + b^2 = c^2$$
$$x^2 + \left(x\sqrt{2}\right)^2 = \left(4\sqrt{3}\right)^2$$
$$x^2 + (x^2)(\sqrt{4}) = (16)(\sqrt{9})$$
$$x^2 + 2x^2 = 48$$
$$3x^2 = 48$$
$$x^2 = 16$$
$$x = 4$$



Therefore, $V = s^3 = (4)^3 = 64 \text{ inches}^3$.
Note: The base of the internal triangle formed is the hypotenuse of a right-isosceles triangle. Also, the answer to $x^2 = 16$ is +4 and –4, but we discard the –4 because distance cannot be negative.

83. Cornered ( )

Choice C
Classification: Plane Geometry Problem
Snapshot: This problem combines circle, square, and triangle geometry. Often the key to calculating the area of the odd-ball figures lies in subtracting one figure from another.
Here the solution to this problem lies in subtracting the area of the smaller (inner) circle from the area of the smaller (inner) square.

i) Area of Outer Square:

$$A = s^2$$
$$2 = s^2$$
$$s = \sqrt{2}$$

ii) Area of Inner Square:

$$A = s^2$$
$$A = (1)^2 = 1 \text{ unit}^2$$

Note that above we pick the number 1 in so far as it is the simplest of integers.

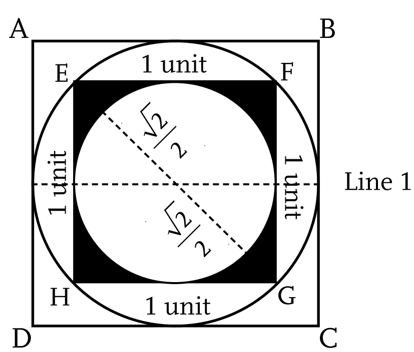
iii) Area of Inner Circle:

$$A = \pi r^2 = \pi(\tfrac{1}{2})^2 = \tfrac{1}{4}\pi \text{ units}^2$$

iv) Area of darkened corners equals Area Inner Square – Inner Circle:

$$A = 1 - \tfrac{1}{4}\pi \text{ units}^2$$

Explanation:

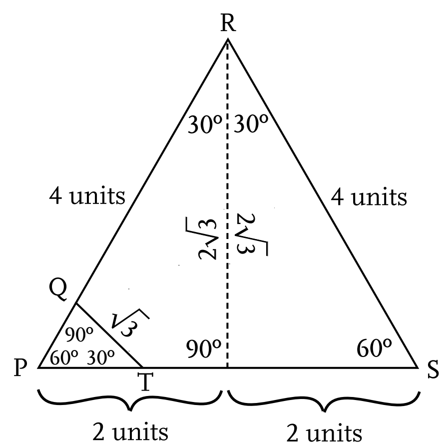


The key to this problem lies in first finding the length of one side of square ABCD. Obviously, if the area of ABCD is 2 square units, the length of one side of square ABCD is calculated as the square root of 2 or $\sqrt{2}$. Line AB equals $\sqrt{2}$, and therefore Line 1 is also $\sqrt{2}$. Line 1 may be viewed as the diameter of the outer circle. It is also the diagonal of square EFGH. EG also equals $\sqrt{2}$, and therefore EH and HG equal 1 unit (because the ratios of the length of the sides in an isosceles right triangle with angle measures of 45°–45°–90° is $1 - 1 - \sqrt{2}$. We can now calculate the measure of square EFGH (where each side equals 1 unit) and the area of the inner circle (with its radius of $\frac{1}{2}$ unit). That is, the length of a side of the inner square equals the diameter of the inner circle. The diameter is twice the radius or, more directly, the radius is one-half the diameter.

84. Woozy ( )

Choice C
Classification: Plane Geometry Problem
Snapshot: This problem provides a review of both equilateral triangles and 30°–60°–90° triangles.

Conceptually, we want to subtract the area of the smaller triangle PQT from the area of the larger equilateral triangle PRS. Note that in a 30°–60°–90° triangle, the ratios of the lengths of the sides are $1:\sqrt{3}:2$ units.



Area of triangle PRS equals:

$$A = \frac{bh}{2} = \frac{4 \times 2\sqrt{3}}{2} = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$$

Area of triangle PQT equals:

$$A = \frac{bh}{2} = \frac{1 \times \sqrt{3}}{2} = \frac{\sqrt{3}}{2} = \tfrac{1}{2}\sqrt{3}$$

Therefore:

$$4\sqrt{3} - \tfrac{1}{2}\sqrt{3} = \tfrac{8}{2}\sqrt{3} - \tfrac{1}{2}\sqrt{3} = \tfrac{7}{2}\sqrt{3}$$

85. Sphere ( )

Choice D
Classification: Solid Geometry Problem
Snapshot: A number of very difficult geometry problems can be solved by picking small manageable numbers.

The best approach is to pick and substitute numbers. Say, for example, that the radius of the original sphere is 2 units, then:

i) Original Sphere:

$$\text{Volume} = \tfrac{4}{3}\pi r^3$$

$$\text{Example } V = \tfrac{4}{3}\pi(2)^3 = \tfrac{32}{3}\pi \text{ units}^3$$

ii) New Sphere:

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$\text{Example } V = \frac{4}{3}\pi(4)^3 = \frac{256}{3}\pi \text{ units}^3$$

iii) Final Calculation:

$$\frac{\text{New}}{\text{Original}} = \frac{\frac{256}{3}\cancel{\pi} \text{ units}^3}{\frac{32}{3}\cancel{\pi} \text{ units}^3} = \frac{\cancel{256}^{\,8}}{\cancel{3}^{\,1}} \times \frac{\cancel{3}^{\,1}}{\cancel{32}^{\,1}} = 8 \text{ times}$$

86. Exam Time ( )

Choice E
Classification: Probability Problem
Snapshot: The probability of two non-mutually exclusive events A or B occurring is calculated by adding the probability of the first event to the second event and then subtracting out the overlap between the two events. This is referred to in probability as the General Addition Rule.

$$\frac{9}{12} + \frac{8}{12} - \frac{6}{12} = \frac{11}{12}$$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. The probability of passing the first exam is added to the probability of passing the second exam, less the probability of passing both exams. If we don't make this subtraction, we will over count because we have overlap—the possibility that she will pass both exams. In this case, we can calculate the overlap as $\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$ because we assume the two events are independent.

Note that situations involving A or B do not necessarily preclude the possibility of both A and B. If we simply add probabilities we inadvertently double count the probability of A and B. We should only count it once and therefore we must subtract it out. See problems 18–20, pages 51–52. One way to prove this result is to recognize that the probability of passing either exam is everything other than failing both exams. The probability of failing both exams is $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$. Therefore, the probability of passing either exam is $1 - \frac{1}{12} = \frac{11}{12}$.

87. Orange & Blue ( )

Choice A
Classification: Probability Problem
Snapshot: This problem introduces the Complement Rule of Probability (refer to Probability Rule 6, page 42).

The best way to view this problem is in terms of what you don't want. At least one orange marble means anything but a blue marble.

Probability of double blue:

$$\frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$$

Therefore, probability of getting at least one orange is the same as one minus the probability of not getting any orange. And the probability of not getting any orange is the same as the probability of getting double blue.

$$P(A) = 1 - P(\text{not } A)$$

$$1 - \frac{3}{10} = \frac{7}{10}$$

The direct method unfolds as follows:

$$\left.\begin{array}{ll} \text{Orange, Blue:} & \frac{2}{5} \times \frac{3}{4} = \frac{6}{20} \\ \text{Blue, Orange:} & \frac{3}{5} \times \frac{2}{4} = \frac{6}{20} \\ \text{Orange, Orange:} & \frac{2}{5} \times \frac{1}{4} = \frac{2}{20} \\ \text{Blue, Blue:} & \frac{3}{5} \times \frac{2}{4} = \frac{6}{20} \end{array}\right\} \frac{14}{20} \Rightarrow \frac{7}{10}$$

Note that the total of all of the above possibilities equals 1 because there are no other possibilities other than the four presented here.

88. Antidote ( )

Choice D
Classification: Probability Problem
Snapshot: This problem type may be called a "kill problem." The key is to solve the problem from the viewpoint of what fraction of the population is still alive. The number killed will be 1 minus the number that is alive.

i) How many people will be alive after three full days?

Answer: $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$

ii) How many people will be dead after three full days?

$$P(A) = 1 - P(\text{not } A)$$

$$1-\frac{8}{27}=\frac{19}{27}$$

The trap answer is: $\left(\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}\right)=\frac{1}{27}$. It is not possible to multiply by $\frac{1}{3}$ because this represents people killed. Everything must first be expressed in terms of how many people are alive. In "kill problems" we can never multiply "dead times dead."

89. Sixth Sense ( )

Choice C
Classification: Probability Problem
Snapshot: This probability problem is made more difficult with the use of the word "exactly." GMAT probability problems which employ the word "exactly" can usually be solved by trial-and-error method. This means simply writing out (or visualizing) all the possibilities.

Second Roll

First Roll		1	2	3	4	5	6
	1	1,1	1,2	1,3	1,4	1,5	**1,6**
	2	2,1	2,2	2,3	2,4	2,5	**2,6**
	3	3,1	3,2	3,3	3,4	3.5	**3,6**
	4	4,1	4,2	4,3	4,4	4,5	**4,6**
	5	5,1	5,2	5,3	5,4	5,5	**5,6**
	6	**6,1**	**6,2**	**6,3**	**6,4**	**6,5**	6,6

As seen in the chart, there are of course thirty-six possible outcomes when we toss a pair of dice (or equally if we role a single die twice). With respect to how we can get exactly one six, we have 10 outcomes: (6,1), (6,2), (6,3), (6,4), (6,5) and (1,6), (2,6), (3,6), (4,6), (5,6). See numbers in bold below.

Let's assume that this problem had asked, "What is the probability of rolling two normal six-sided dice and getting at least one six?" The correct answer would have been choice D.

In solving this particular problem, we would employ the General Addition Rule:

$$P(A \text{ or } B)=P(A)+P(B)-P(A \text{ and } B)$$

$$\frac{1}{6}+\frac{1}{6}-\frac{1}{36}=\frac{12}{36}-\frac{1}{36}=\frac{11}{36}$$

Note that we can't just add one-sixth and one-sixth to get twelve over thirty-six or one-third, which incidentally, is Answer choice E. To do so would fail to account for and properly remove the double overlap created when double sixes are rolled.

One further way to confirm our answer is through the use of the Complement Rule. The probably of rolling <u>at least</u> one six is the same as the probability of *one minus the probability of rolling no sixes.*

$$1-\left(\frac{5}{6}\times\frac{5}{6}\right)=\frac{36}{36}-\frac{25}{36}=\frac{11}{36}$$

Anyway, the point is that this provides an excellent jumping off point to solve the original, "What is the probability of rolling two normal six-sided dice and getting exactly one six?" All we need to do is to subtract out $\frac{1}{36}$ from the previous calculation (i.e., $\frac{11}{36}$) in order to remove the probability of rolling double sixes. Note that in the calculation below, the "first" six is removed because it's overlap while the "second" six is removed because it represents the probability of rolling double sixes.

$$\frac{1}{6}+\frac{1}{6}-\frac{1}{36}-\frac{1}{36}=\frac{10}{36}=\frac{5}{18}$$

Author's note: Some candidates wonder if there is a "real" mathematical way to solve these "exactly-type" problems. The answer is yes—it involves *combinations in binomials*. Such probability theory is beyond the scope of this book and, fortunately, it is not something that a person would be expected to know for the purposes of taking the GMAT. Nevertheless, the formula and solution are included here should readers desire to research the topic further.

$$P=\left({}_{n}C_{r}\right)q^{n-r}p^{r} \quad p = \text{probability of the event desired; } q = \text{probability of event not desired}$$

$$P=\left({}_{2}C_{1}\right)q^{n-r}p^{r} \quad {}_{2}C_{1} = \text{combinations of two numbers in which one is a six}$$

$$P=\left(\frac{n!}{r!(n-r)!}\right)q^{n-r}p^{r}$$

$$P=\left(\frac{2!}{1!(2-1)!}\right)\left(\frac{5}{6}\right)^{2-1}\left(\frac{1}{6}\right)^{1}$$

$$P=\left(\frac{2\times\cancel{1}}{\cancel{1\times(1)!}}\right)\left(\frac{5}{6}\right)^{1}\left(\frac{1}{6}\right)^{1}$$

$$P=2\left(\frac{5}{36}\right)$$

$$P=\frac{10}{36}$$

$$P=\frac{5}{18}$$

For summary purposes, let's try one more example. Suppose a question asks: "A coin is tossed three times. What is the probability of getting exactly one head?"

Again, GMAT problems involving the word "exactly" can usually be solved by the "trial-and-error" method. This means simply writing out all the possibilities. For instance, we know that there are eight possibilities and that each of the eight outcomes has exactly a one-eighth chance of occurring.

Example: $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

HHH	TTT
HHT	TTH
HTH	THT
HTT	**THH**

We can easily confirm the answer by adding up those outcomes in which we have exactly two heads: HHT, HTH, and THH. Three occurrences out of eight are possible. The answer becomes $\frac{3}{8}$ or 37.5%. For the record, here is the solution to this problem using the formula for *combinations in binomials:*

$P = \left({}_nC_r\right) q^{n-r} p^r$ p = probability of the event desired; q = probability of event not desired

$P = \left({}_3C_1\right) q^{n-r} p^r$ ${}_3C_1$ = combinations of three events in which one is a head

$$P = \left(\frac{n!}{r!(n-r)!}\right) q^{n-r} p^r$$

$$P = \left(\frac{3!}{1!(3-1)!}\right)\left(\frac{1}{2}\right)^{3-1}\left(\frac{1}{2}\right)^1$$

$$P = \left(\frac{3 \times \cancel{2 \times 1}}{\cancel{1 \times (2)!}}\right)\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1$$

$$P = 3\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$P = 3\left(\frac{1}{8}\right)$$

$$P = \frac{3}{8}$$

What if the question had asked: "A coin is tossed three times. What is the probability of getting <u>at least</u> one head?"

The answer would be $\frac{7}{8}$ or 87.5%. We can easily confirm this result by adding up those outcomes in which we get at least one head: HHH, HHT, HTH, HTT, TTH, THT, and THH. Note that this answer represents everything except "triple tails" (i.e., TTT). A quicker way to solve this problem would be to use the Complement Rule. "One minus the probability of getting all tails" would also give us "at least one head."

$$1 - \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) = \frac{8}{8} - \frac{1}{8} = \frac{7}{8}$$

Finally, what if the question had asked: "A coin is tossed three times. What is the probability of getting at most one head?"

The answer would be $\frac{4}{8}$ or 50%. We can easily confirm this result by adding up those outcomes in which we get at most one head: HTT, TTH, THT, and TTT. Note that this answer also includes the possibility of getting all tails (i.e., TTT).

90. At Least One ( )

Choice E
Classification: Probability Problem
Snapshot: This problem involves three overlapping probabilities and is best solved using the Complement Rule of Probability. The direct mathematical approach is more cumbersome, but parallels the solution to *German Cars,* problem 20 in this chapter.

I. Shortcut Approach

Using the Complement Rule, the probability of total failure is calculated as one minus the probability of failing all three exams:

i) The probability of *not* passing the first exam:

$$P(\text{not } A) = 1 - P(A) \qquad 1 - \frac{3}{4} = \frac{1}{4}$$

ii) Below is the probability of *not* passing the second exam:

$$P(\text{not } B) = 1 - P(B) \qquad 1 - \frac{2}{3} = \frac{1}{3}$$

iii) Below is the probability of *not* passing the third exam:

$$P(\text{not } C) = 1 - P(C) \qquad 1 - \frac{1}{2} = \frac{1}{2}$$

iv) The probability of failing *all* three exams:

$$P(\text{not } A \text{ or } B \text{ or } C) \qquad \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} = \frac{1}{24}$$

v) The probability of passing *at least* one exam:

$$P(A) = 1 - P(\text{not } A) \qquad 1 - \frac{1}{24} = \frac{23}{24}$$

II. Direct Approach

The direct method is much more cumbersome. Mathematically it is solved by calculating the probability of passing only one of the three exams, two of the three exams, and all of the three exams.

1. Probability of passing exam one but *not* exams two or three:

   $P(A) \times P(\text{not } B) \times P(\text{not } C) \qquad \frac{3}{4} \times \frac{1}{3} \times \frac{1}{2} = \frac{3}{24}$

2. Probability of passing exam two but *not* exams one or three:

   $P(\text{not } A) \times P(B) \times P(\text{not } B) \qquad \frac{1}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{2}{24}$

3. Probability of passing exam three but *not* exams one or two:

   $P(\text{not } A) \times P(\text{not } B) \times P(C) \qquad \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} = \frac{1}{24}$

4. Probability of passing exams one and two but *not* exam three:

   $P(A) \times P(B) \times P(\text{not } C) \qquad \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{6}{24}$

5. Probability of passing exams one and three but *not* exam two:

   $P(A) \times P(\text{not } B) \times P(C) \qquad \frac{3}{4} \times \frac{1}{3} \times \frac{1}{2} = \frac{3}{24}$

6. Probability of passing exams two and three but *not* exam one:

   $P(\text{not } A) \times P(B) \times P(C) \qquad \frac{1}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{2}{24}$

7. Probability of passing all three exams:

   $P(A) \times P(B) \times P(C) \qquad \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{6}{24}$

8. Probability of not passing any of the three exams:

   $P(\text{not } A) \times P(\text{not } B) \times P(\text{not } C) \qquad \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} = \frac{1}{24}$

The above are all the possibilities regarding the outcomes of one student taking three exams. Adding the first seven of eight possibilities above will result in the correct answer using the direct approach.

Proof: $\frac{3}{24} + \frac{2}{24} + \frac{1}{24} + \frac{6}{24} + \frac{3}{24} + \frac{2}{24} + \frac{6}{24} = \frac{23}{24}$

Note that the total of all eight outcomes above will total to 1 because 1 is the sum total of all probabilistic possibilities.

Proof: $\frac{3}{24} + \frac{2}{24} + \frac{1}{24} + \frac{6}{24} + \frac{3}{24} + \frac{2}{24} + \frac{6}{24} + \frac{1}{24} = \frac{24}{24} = 1$

91. Coin Toss ( )

Choice D
Classification: Probability Problem
Snapshot: To highlight how the words "at most" also trigger the Complement Rule of Probability.

This easiest way to do this problem is to think in terms of what we don't want. At most three heads means that we want anything except all heads or four heads. This includes the following:

We don't want all heads: HHHHH $\frac{1}{32}$

We don't want four heads:
HHHHT $\frac{1}{32}$
HHHTH $\frac{1}{32}$
HHTHH $\frac{1}{32}$
HTHHH $\frac{1}{32}$
THHHH $\frac{1}{32}$

$\frac{5}{32}$

$\frac{6}{32}$

Thus, $1-\left(\frac{1}{32}+\frac{5}{32}\right)=1-\frac{6}{32}=\frac{26}{32}=\frac{13}{16}$

Author's note: This problem is the mirror opposite of the very problem which states, "If a coin is tossed five times what is the probability of heads appearing at least two times?"

We don't want all tails: TTTTT $\frac{1}{32}$

We don't want just one head:
HTTTT $\frac{1}{32}$
THTTT $\frac{1}{32}$
TTHTT $\frac{1}{32}$
TTTHT $\frac{1}{32}$
TTTTH $\frac{1}{32}$

$\frac{5}{32}$

$\frac{6}{32}$

Thus, $1-\left(\frac{1}{32}+\frac{5}{32}\right)=1-\frac{6}{32}=\frac{26}{32}=\frac{13}{16}$

92. Hiring ( )

Choice B
Classification: Permutation Problem (Noted Exception)
Snapshot: This particular problem falls under neither the umbrella of probability nor permutation nor combination. It is included here because it is so frequently mistaken for a permutation problem.

$$7 \times 4 \times 10 = 280$$

The solution requires only that we multiply together all individual possibilities. Multiplying 7 (candidates for sales managers) by 4 (candidates for shipping clerk) by 10 (candidates for receptionist) would result in 280 possibilities.

Author's note: This problem is about a series of choices. It utilizes the "multiplier principle" and falls within the Rule of Enumeration. The permutation formula cannot be used with this type of problem. This problem is concerned with *how many choices we have,* not *how many arrangements are possible,* as is the case with a permutation problem.

93. Fencing ( )

Choice C
Classification: Permutation Problem
Snapshot: This problem is a permutation, not a combination, because order does matter. If country A wins the tournament and country B places second, it is a different outcome than if country B wins and country A places second.

$$_{n}P_{r} = \frac{n!}{(n-r)!}$$

$$_{4}P_{2} = \frac{4!}{(4-2)!} = \frac{4 \times 3 \times \cancel{2 \times 1}}{\cancel{2!}} = 12$$

Author's note: Consider this follow-up problem. A teacher has four students in a special needs class. She must assign four awards at the end of the year: math, English, history, and creative writing awards. How many ways could she do this assuming that a single student could win multiple awards?

$$n^r = 4^4 \qquad 4 \times 4 \times 4 \times 4 = 256$$

She has four ways she could give out the Math award, four ways she could give out the English award, four ways to give out the History award, and four ways to give out the Creative Writing award. Refer to Probability Rule 10, page 43.

94. Alternating ( )

Choice C
Classification: Permutation Problem
Snapshot: This problem is foremost a joint permutation, in which we calculate two individual permutations and multiply those outcomes together. This problem also incorporates the “mirror image” rule of permutations.

There are two possibilities with respect to how the girls and boys can sit for the make-up exam. A boy will sit in the first, third, and fifth seats and a girl will sit in the second, fourth, and sixth seats or a girl will sit in the first, third, and fifth seats and a boy will sit in the second, fourth, and sixth seats.

Scenario 1		**Scenario 2**
B G B G B G		G B G B G B
$\frac{3}{B_1} \times \frac{3}{G_1} \times \frac{2}{B_2} \times \frac{2}{G_2} \times \frac{1}{B_3} \times \frac{1}{G_3}$	*or*	$\frac{3}{G_1} \times \frac{3}{B_1} \times \frac{2}{G_2} \times \frac{2}{B_2} \times \frac{1}{G_3} \times \frac{1}{B_3}$

With reference to the scenario 1 above, how many ways can each seat be filled (left to right)? Answer: The first seat can be filled by one of three boys, the second seat can be filled by one of three girls, the third seat can filled by one of two remaining boys, the fourth seat can be filled by one of two remaining girls, the fifth seat will be filled by the final boy, and the sixth seat will be filled by the final girl.

With reference to the scenario 2 above, how many ways can each seat be filled (left to right)? Answer: The first seat can be filled by one of three girls, the second seat can be filled by one of three boys, the third seat can filled by one of two remaining girls, the fourth seat can be filled by one of two remaining boys, the fifth seat will be filled by the final girl, and the sixth seat will be filled by the final boy.

Therefore:

$$(3 \times 3 \times 2 \times 2 \times 1 \times 1) + (3 \times 3 \times 2 \times 2 \times 1 \times 1)$$
$$36 + 36 = 72$$

In short, the answer is viewed as:

$$(3! \times 3!) + (3! \times 3!)$$
$$2(3! \times 3!)$$
$$2[(3 \times 2 \times 1) \times (3 \times 2 \times 1)] = 72$$

Author's note: There are two common variations stemming from this type of permutation problem:

i) Three boys and three girls are going to sit for a make-up exam. The girls are to sit in the first, second, and third seats while the boys must sit in the fourth, fifth, and sixth seats. How many possibilities are there with respect to how the six students can be seated?
Answer: $3! \times 3! = 6 \times 6 = 36$ possibilities

ii) Three boys and three girls are going to sit for a make-up exam. If there are no restrictions on how the students may be seated, how many possibilities are there with respect to how they can be seated?

Answer: $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ possibilities

95. Banana ( )

Choice E
Classification: Permutation Problem

Snapshot: This problem highlights the handling of "repeated letters" (or "repeated numbers"). The formula for calculating permutations with repeated numbers or letters is $\frac{n!}{x!y!z!}$, where x, y, and z are distinct but identical numbers or letters.

$$\frac{n!}{x!y!} = \frac{6!}{3! \times 2!} = \frac{6 \times 5 \times \cancel{4}^{2} \times \cancel{3 \times 2 \times 1}}{(\cancel{3 \times 2 \times 1}) \times (\cancel{2 \times 1})} = 60$$

The word "banana" has three "a's" and two "n's."

96. Table ( )

Choice C
Classification: Permutation Problem
Snapshot: This problem deals with the prickly issue of "empty seats."

$$\frac{5!}{2!} = \frac{5 \times 4 \times 3 \times \cancel{2!}}{\cancel{2!}} = 60$$

Author's note: The answer to this problem is similar in approach to that of the previous problem, *Banana*. In permutation theory, "empty seats" are analogous to "identical numbers" (or "identical letters").

Also the geometric configuration of a table should not mislead. The solution to this problem would be identical had we been dealing with a row of five seats.

97. Singer ( )

Choice C
Classification: Combination Problem
Snapshot: Joint Combinations are calculated by multiplying the results of two individual combinations.

First, break the combination into two calculations. First, the "old songs," and second, the "new songs."

$$_{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$_{6}C_{4} = \frac{6!}{4!(6-4)!} = \frac{6!}{4!(2)!} = \frac{\cancel{6}^{3} \times 5 \times \cancel{4 \times 3 \times 2 \times 1}}{\cancel{4 \times 3 \times 2 \times 1} \times (\cancel{2 \times 1})} = 15$$

Thus, 15 represents the number of ways the singer can choose to sing four of six songs.

$$_{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$_{5}C_{2} = \frac{5!}{2!(5-2)!} = \frac{5!}{2!(3)!} = \frac{5 \times \cancel{4}^{2} \times \cancel{3 \times 2 \times 1}}{\cancel{2 \times 1}\,(\cancel{3 \times 2 \times 1})} = 10$$

Thus, 10 represents the number of ways the singer could chose to sing three of five old songs. Therefore, the joint combination equals $15 \times 10 = 150$.

In summary, the outcome of this joint combination is:

$$_{n}C_{r} \times {}_{n}C_{r} = \frac{n!}{r!(n-r)!} \times \frac{n!}{r!(n-r)!}$$

$$_{6}C_{4} \times {}_{5}C_{2} = \frac{6!}{4!(6-4)!} \times \frac{5!}{2!(5-2)!} = 15 \times 10 = 150$$

98. Outcomes ( )

Choice A
Classification: Combination Problem
Snapshot: This bonus problem exists to test permutation and combination theory at a grass roots level. A two-chili rating is assigned because it is meant to be completed within two minutes (the average time allocated for a GMAT math problem). A strong understanding of theory will allow the test taker to avoid doing any calculations.

Statement I:

True. $_5P_3 > {_5P_2}$

$_5P_3 = 60$ and $_5P_2 = 20$. Order matters in permutations and more items in a permutation equals more possibilities.

Statement II:

False. $_5C_3 > {_5C_2}$

$_5C_3 = 10$ and $_5C_2 = 10$. Strange as it may seem, the outcomes are equal! "Complements in combinations" result in the same probability. Complements occur when the two inside numbers equal the same outside number. Here $3 + 2 = 5$. Note this phenomenon only occurs in combinations, not permutations.

Statement III:

False. $_5C_2 > {_5P_2}$

$_5C_2 = 10$ and $_5P_2 = 20$. Order matters in permutations and this creates more possibilities relative to combinations. Stated in the reverse, order doesn't matter in combinations and this results in fewer outcomes than permutations, all things being equal.

99. Reunion ( )

Choice D
Classification: Combination Problem
Snapshot: This problem *Reunion* is a rather complicated sounding problem but its solution is actually quite simple. We're essentially asking: How many groups of two can we create from eleven items where order doesn't matter? Or how many ways can we choose two items from eleven items where order doesn't matter?

$$_nC_r = \frac{n!}{r!(n-r)!}$$

$$_{11}C_2 = \frac{11!}{2!(11-2)!} = \frac{11 \times 10 \times \cancel{9!}}{2!(\cancel{9!})} = 55$$

100. Display ( )

Choice B
Classification: Combination Problem
Snapshot: This problem combines both the combination formula and probability theory.

First, the total number of ways she can choose 3 computers from 8 is represented by the following combination.

$$ {}_{n}C_{r} = \frac{n!}{r!(n-r)!} $$

$$ {}_{8}C_{3} = \frac{8!}{3!(8-3)!} = \frac{8 \times 7 \times 6 \times \cancel{5!}}{3!(\cancel{5!})} = 56 $$

Second, the total number of ways in which the two most expensive computers will be among the three computers is 6. For example, one way to visualize the situation is to think of the eight computers as A, B, C, D, E, F, G, and H. If A and B are the most expensive computers, then there are six ways these two computers could be among the three computers chosen, namely ABC, ABD, ABE, ABF, ABG, and ABH. Yet another way of arriving at this figure is to visualize the two most expensive computers as fixed within any group of three. Therefore, we ask "How many ways can we choose a final computer from the group of eight, given that A and B are already in our group?" The answer to this part of the problem is derived by the following combination.

$$ {}_{6}C_{1} = \frac{6!}{1!(6-1)!} = \frac{6 \times 5!}{1!(5!)} = 6 $$

The final answer: $\frac{6}{56} = \frac{3}{28}$

In summary, the following is perhaps the most succinct way to view this problem:

$$ \frac{{}_{n}C_{r}}{{}_{n}C_{r}} = \frac{{}_{6}\mathrm{C}_{1}}{{}_{8}\mathrm{C}_{3}} = \frac{6}{56} = \frac{3}{28} $$

Below is an alternative solution:

$$ \frac{1}{8} \times \frac{1}{7} \times 6 = \frac{6}{56} = \frac{3}{28} $$

ABOUT THE AUTHOR

A graduate of the University of Chicago's Booth School of Business and certified public accountant, Brandon first developed an expertise in GMAT test-taking and MBA admissions strategies while working overseas for the world's largest test-preparation organization. This book represents his distilled experience gained from classroom teaching on two continents and individual tutor sessions that have helped hundreds of applicants beat the GMAT and achieve acceptance at the world's leading business schools.

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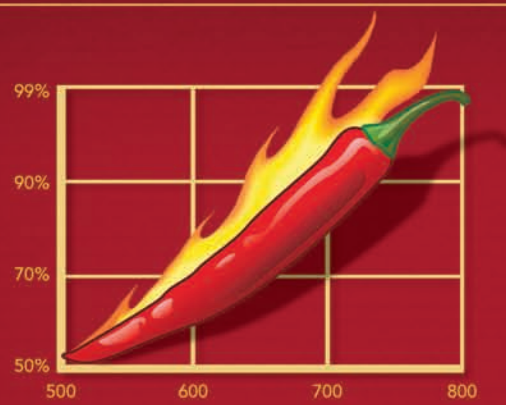

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